Quality, Speed and Variability

- **Process/Product Excellence**
- **Optimized Process Flow**
- **Consequences of Variability**

Prof. Krishna R. Pattipati
UTC Professor in Systems Engineering
Dept. of Electrical and Computer Engineering
University of Connecticut
Contact: krishna@engr.uconn.edu (860) 486-2890

January 24, 2013

Overview

- Slogan: Lean versus Six Sigma
- Quality Engineering
  - Off-line Design for Quality and On-line Quality Control
  - Goal: Variability Reduction
  - Process Capability
- Basic Factory Dynamics
  - Factory: A *goal-oriented network* of *processes* through which *parts flow*
  - Little's Theorem linking Work in Process (WIP, Queue length) with Cycle time (Response time) and Throughput
  - Variability makes a difference here as well

2

Lean versus Six Sigma

- Lean focuses on cycle time reduction
  - Cycle time = value-added time + non-value added time
  - 5S: sort, straighten, scrub, standardize, sustain
- Six Sigma focuses on reducing variability, thereby improving product/process quality
  - Widely used in industry (Motorola, GE, P&W, Boeing…)

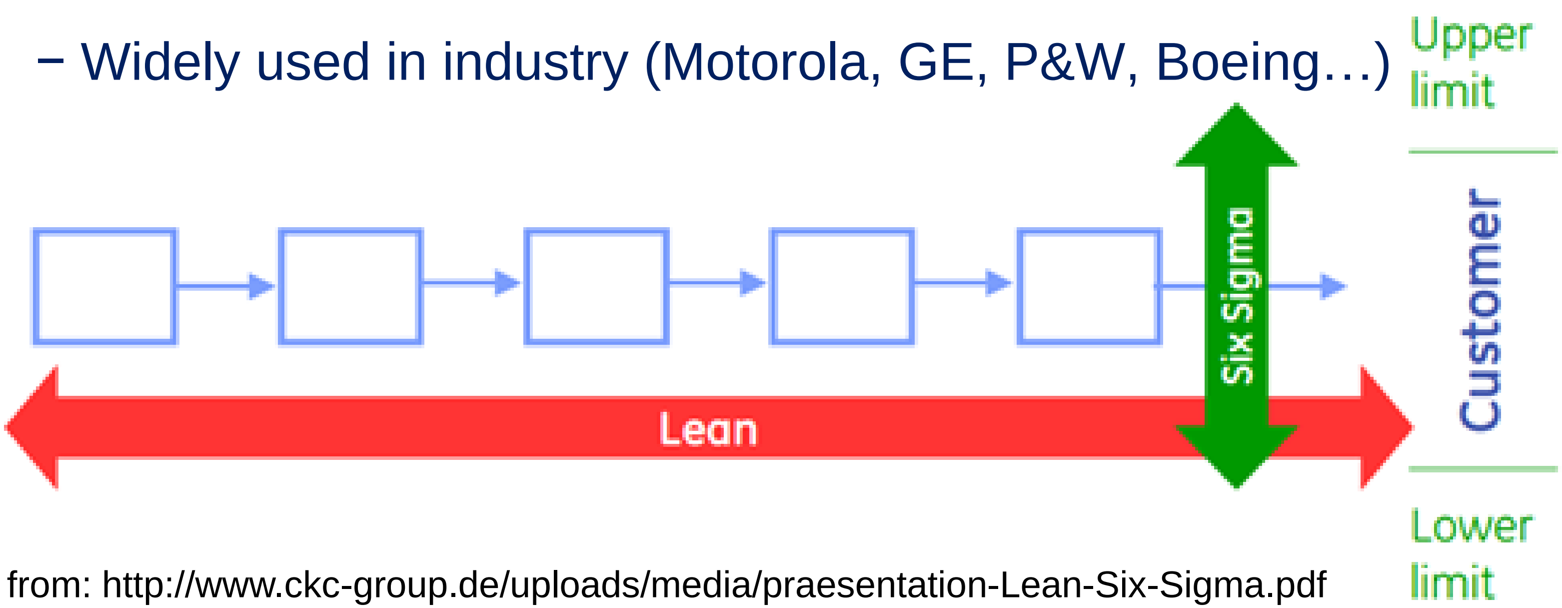


Figure from: http://www.ckc-group.de/uploads/media/praesentation-Lean-Six-Sigma.pdf

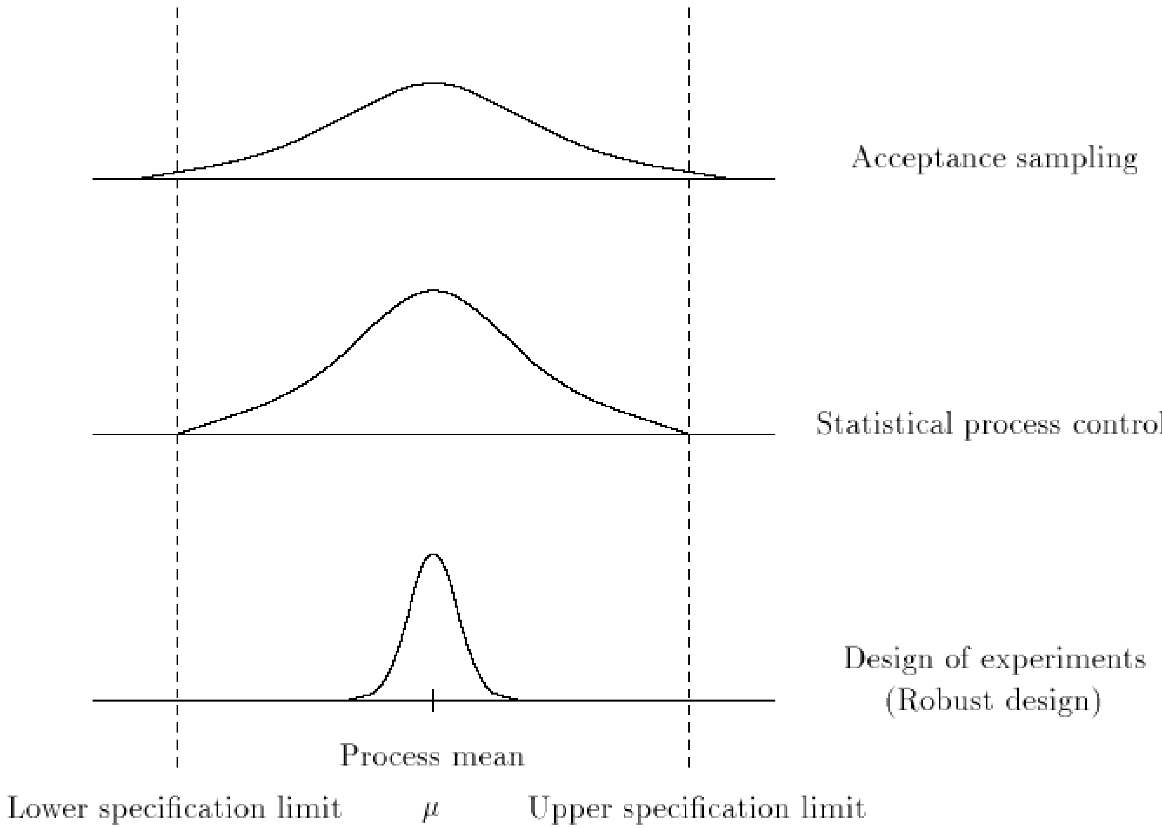
Why Quality?

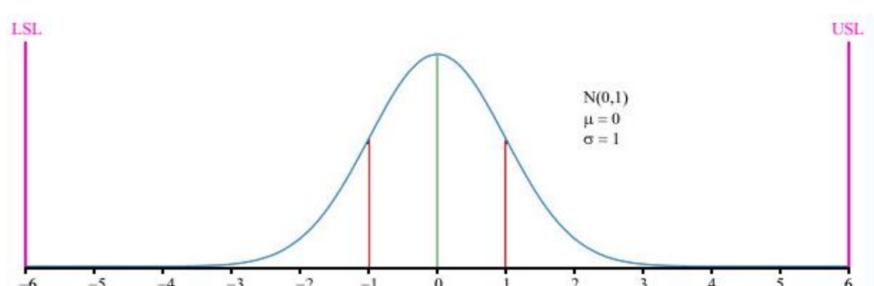
- Quality: key to economic success
  - – increase in productivity at little cost
  - – vital for business growth and enhanced competitive position
- Cost of fixing problems in the field increases exponentially!

LEVEL OF ASSEMBLY	COST PER FAILURE ($)
COMPONENT LEVEL	1
CIRCUIT BOARD LEVEL	10
BOX LEVEL	100
SYSTEM LEVEL	1000
FIELD OPERATION LEVEL	2000-20,000

Latest Example: Boeing 787 grounded for Li-ion Battery Problems

The Goal: Variability Reduction





Six sigma ⇒ 3.4 parts in a Million defective and process capability ratio, $CP_k = 2$. Why?

Normal (Gaussian, Bell) pdf and CDF

$$pdf : p_X(x) = N(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

$$CDF : P\{X \le x\} = \Phi(x; \mu, \sigma^2) = \int_{-\infty}^{x} N(y \mid \mu, \sigma^2)\, dy$$

$$\mu = E[X] = \text{mean (and mode)}$$

$$\sigma^2 = E[(X-\mu)^2] = \text{variance}$$

$$\sigma = \text{standard deviation} = \sqrt{\text{variance}} \ge 0$$

$$Z = \frac{X-\mu}{\sigma} \sim N(0,1) = \text{Normalized Gausian random Variable}$$

$$P(|X-\mu| \ge n\sigma) = P\left(\frac{|X-\mu|}{\sigma} \ge n\right) = P(|Z| \ge n)$$

$$= P(Z \le -n) + P(Z \ge n) = 2P(Z \le -n) = 2\Phi(-n; 0,1)$$

n : σ level

Controversial Assumption of Six Sigma!

Six Sigma analysis assumes that mean changes by $\pm 1.5\sigma$ in the long run!

$$\mu_{new} = \mu \pm 1.5\sigma$$

Assume, without loss of generality, $\mu_{new} = \mu + 1.5\sigma$

$$\Rightarrow X \sim N(x \mid \mu_{new}, \sigma^2)$$

Let $Z = \dfrac{X - \mu_{new}}{\sigma}$

$$P(|X-\mu| \geq n\sigma) = P\left(\frac{|X-\mu|}{\sigma} \geq n\right)$$

$$= P\left(\frac{|X-\mu_{new}+\mu_{new}-\mu|}{\sigma} \geq n\right)$$

$$= P(|Z+1.5| \geq n) = P(Z \leq -(n+1.5)) + P(Z \geq n-1.5)$$

$$= P(Z \leq -(n+1.5)) + P(Z \leq -(n-1.5))$$

$$= \Phi(-(n+1.5);0,1) + \Phi(-(n-1.5);0,1) \overset{6\sigma \text{ assumption}}{\approx} \Phi(-(n-1.5);0,1)$$

Defects per Million Opportunities (DPMO)

$\Phi(-(n+1.5);0,1)$
$+\Phi(-(n-1.5);0,1)$

$\Phi(-(n-1.5);0,1)$

n = σ Level	DPMO (if mean does not change)	DPMO (if mean changes by 1.5σ)	DPMO (if mean changes by 1.5σ and one sided 6σ assumption)	Experience-based Cost of Quality
1	317,311	697,672	691,462	NA
2	45,500	308,770	308,538	NA
3	2,700	66,811	66,807	25%-40% of sales
4	63	6,210	6,210	15%-25% of sales
5	0.57	233	233	5%-15% of sales
6	0.002	3.4	3.4	<1% of sales

Free stat calculator at ww.xuru.org/st/PD.asp and many others

Process Capability

- Process capability analysis is an activity involving
  - Quantification of process variability
  - Analysis of process variability relative to product specifications
  - Assists manufacturing in eliminating/reducing variability
- Measure of process capability: customarily the 6-sigma spread in distribution of the product quality characteristic
  - Natural tolerance limits (UNTL and LNTL) of a process

$$\text{UNTL} = \mu + 3\sigma$$
$$\text{LNTL} = \mu - 3\sigma$$

  - Specification limits on the process: USL (upper specification limit) and LSL (lower specification limit)

Process Capability Index (C_{pk})

- Process capability ratio

$$C_{pk} = \frac{USL - LSL}{6\sigma}$$

  - Interpretation $$P = \left(\frac{1}{C_{pk}}\right)100$$

P: percentage of specification band used up by the process

- One-sided specifications (if only either USL or LSL is relevant)

$$C_{pkU} = \frac{USL - \mu}{3\sigma}$$

$$C_{pkL} = \frac{\mu - LSL}{3\sigma}$$

- Process capability for off-centered process: take the one-sided C_{pk} for the specification limit closest to the process average

$$C_{pk} = \min(C_{pkU}, C_{pkL})$$

- Six Sigma: Centered $\Rightarrow C_{pk} = 2$; 1.5σ off-centered $\Rightarrow C_{pk} = 1.5$

Measures of Quality Loss

- Fraction defective = (# of rejects)/(total # of parts)
  - leads to the use of a step loss function in the *tolerance interval*
- Quadratic loss function

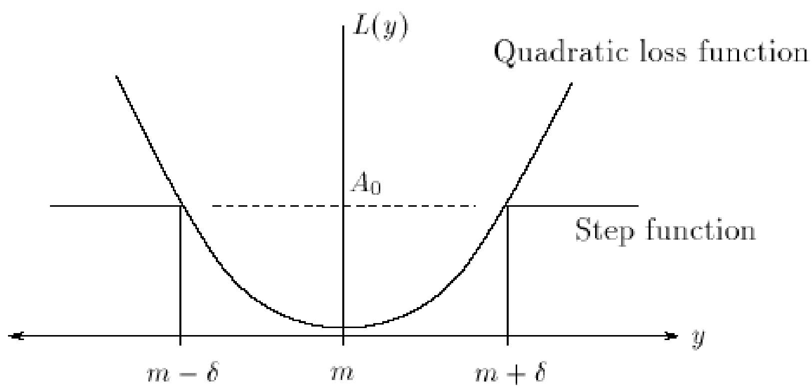
$$L(y) = k(y-m)^2$$

y : quality characteristic

m : target for y

  - results in smaller overall (expected) loss to society:

$$k = \frac{A_0}{\delta^2}$$



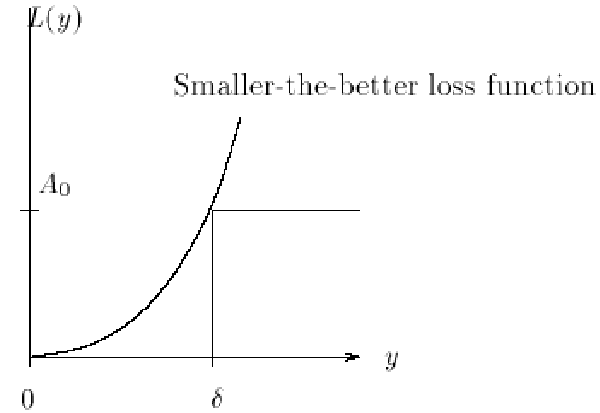
Other Loss Functions

- *Smaller-the-better* type quality characteristic

$$L(y) = ky^2$$

$$k = \frac{A_0}{\delta^2}$$

Example: Number of defects in a composite-material part

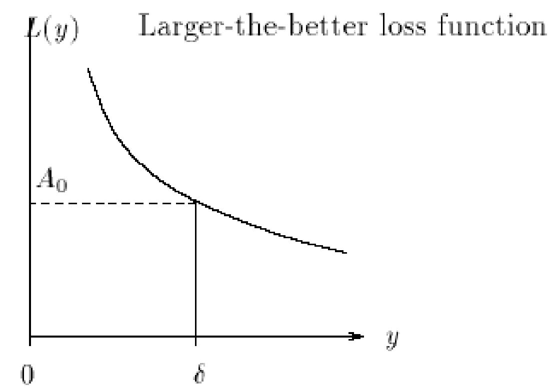


- *Larger-the-better* type quality characteristic

$$L(y) = k\left[\frac{1}{y^2}\right]$$

$$k = A_0\delta^2$$

Example: strength of a part



Robust Design

- An *off-line* design technique: using experiments, find the settings of the product/process parameters (*design parameters*) which minimize sensitivity of the quality characteristic to external/uncontrollable variations (achieve *robustness*) — a.k.a. Taguchi's method

  1. Identify a measure of variability (performance measure) that is a function of the design parameters, e.g., an average loss function, or a *signal-to-noise ratio*
  2. Identify factors (variables) associated with the uncontrollable variations → noise factors — distinct from the design factors
  3. Conduct experiment: systematically vary the design parameters as well as the noise factors to get estimates of the variability measure for chosen set of design parameter settings
  4. Conduct data analysis to get process model and to identify the best design parameter settings (those that minimize the variability measure)
  5. Run verification experiment to ensure that the 'best' design yields the expected improvement

Measure of Variability/Robustness

- Ideal measure for nominal-the-best type characteristic: the *expected quadratic loss function* (average quality loss per product)

$$\text{MSE}(\mathbf{x}) = E[L(y(\mathbf{x}))] = k(\mu(\mathbf{x}) - m)^2 + k\sigma^2(\mathbf{x})$$

  minimize MSE($\mathbf{x}$) with respect to design parameters $\mathbf{x}$

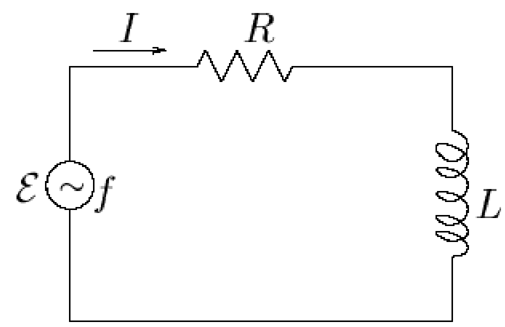
- Taguchi: instead of mimimizing MSE($\mathbf{x}$), maximize a *signal-to-noise ratio*, e.g.,

$$SN_T = 10 \log_{10} \frac{\mu^2}{\sigma^2}$$

  1. Fix levels for each factor, run an experiment at different combinations of factor-levels, measure y and SN_T
  2. Separate out ***signal factors*** from the other design factors (***control factors***)
     - ***Signal factors***: affect the mean μ of the response y but not SN_T
  3. Maximize SN_T with respect to control factors $\overset{?}{\Rightarrow}$ minimize σ
  4. Adjust signal factors to bring mean on target

Scope for Robust Design: AC Circuit Example

- $\mathcal{E}$ at a tightly toleranced frequency f of either 50 or 60 Hz, and an rms value of 100 VAC with a tolerance of $\pm 10\%$
- Design parameters: nominal values of R and L — toleranced at $\pm 10\%$ about nominal, e.g., $3\sigma_{R_0} = 0.1 R_0$



$$I = \frac{\mathcal{E}}{\sqrt{R^2 + (2\pi f L)^2}}$$

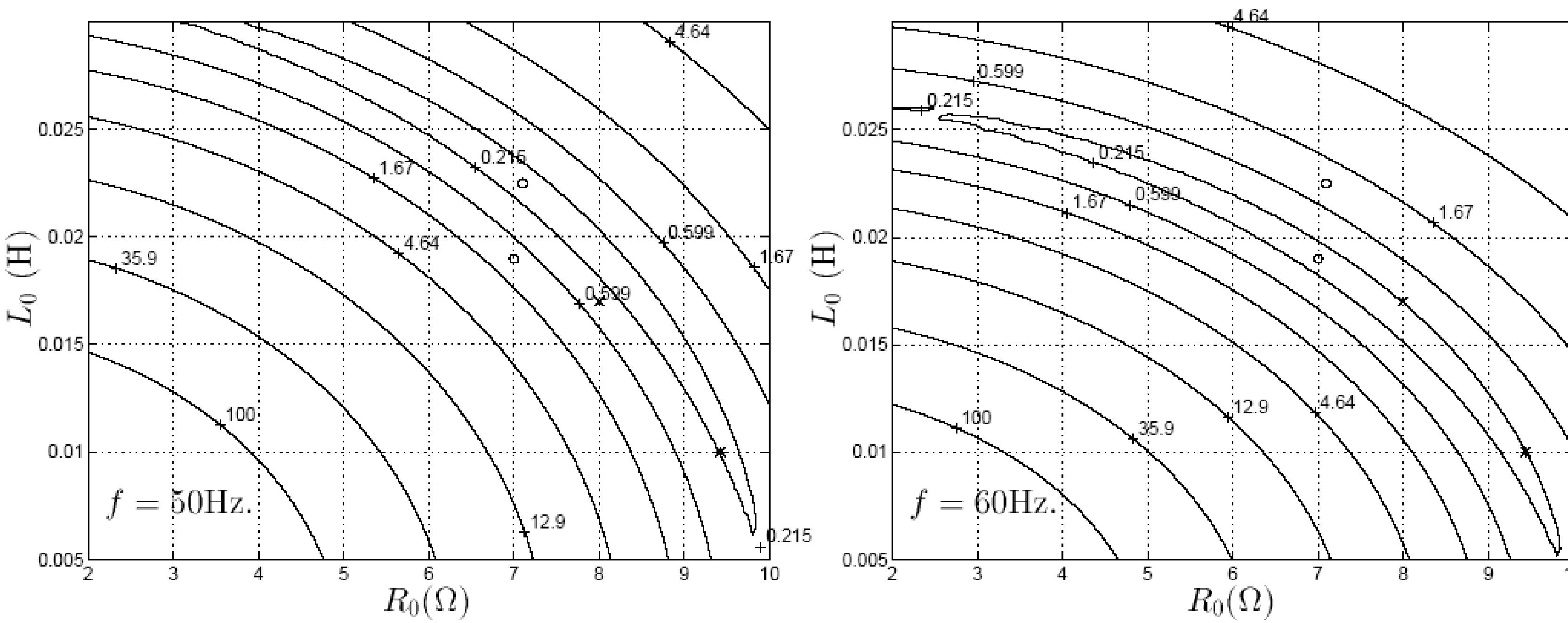
- Objective: Find nominal R and L such that I is as close to 10 amperes and with as little variability as possible
- Minimize the mean squared-error

$$\mathrm{MSE}_I(R_0, L_0) = \sigma_I^2(R_0, L_0) + (\mu_I(R_0, L_0) - T_I)^2$$

$$\mu_I(\mathcal{E}_0, f_0, R_0, L_0) \simeq I_0 + \frac{1}{2}\left(\left.\frac{\partial^2 I}{\partial R^2}\right|_0 \sigma_{R_0}^2 + \left.\frac{\partial^2 I}{\partial L^2}\right|_0 \sigma_{L_0}^2 \right)$$

$$\sigma_I^2(\mathcal{E}_0, f_0, R_0, L_0) \simeq \left[\frac{\partial I}{\partial \mathcal{E}}\right]_0^2 \sigma_{\mathcal{E}_0}^2 + \left[\frac{\partial I}{\partial R}\right]_0^2 \sigma_{R_0}^2 + \left[\frac{\partial I}{\partial L}\right]_0^2 \sigma_{L_0}^2$$

AC Circuit: MSE Contours

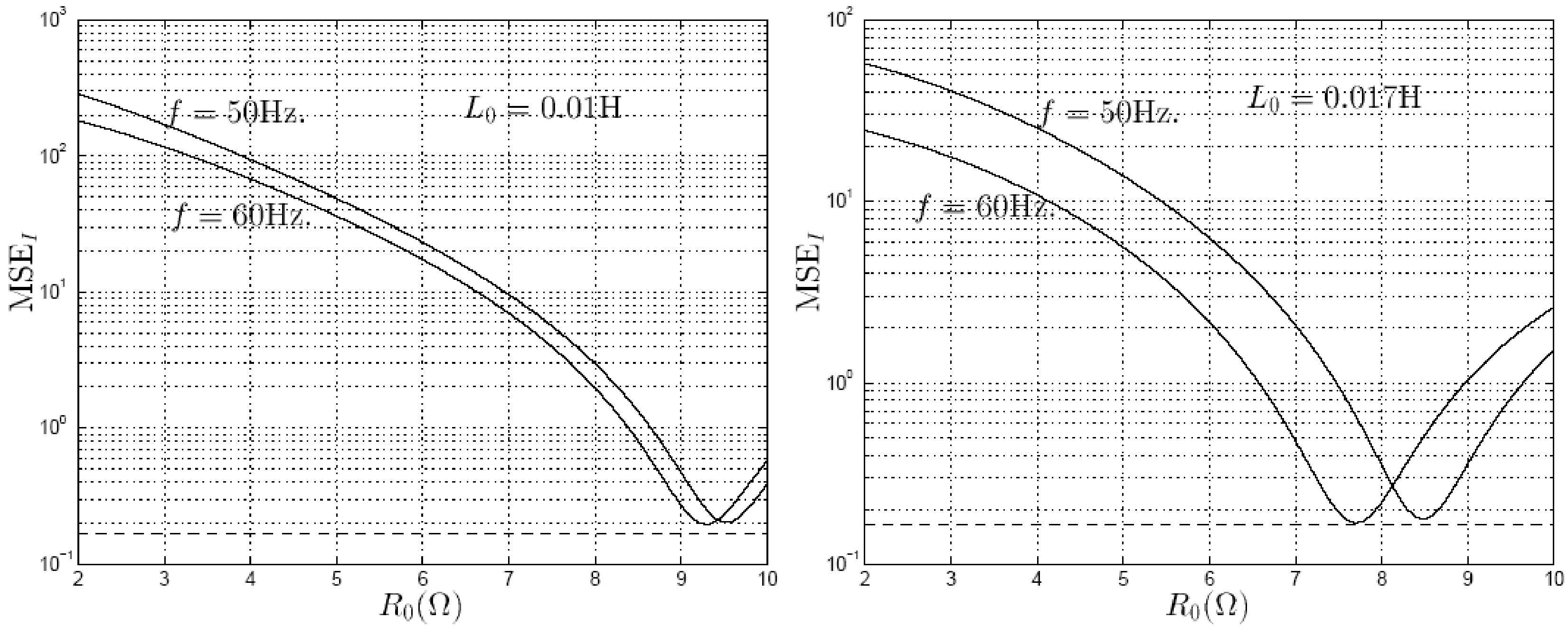


'$*$' final design

'$\times$' design obtained in [Boza, et al (1994)]

'$\circ$' individual optima of MSE_I

AC Circuit: MSE Profiles



AC Circuit: Performance Summary

R_0 (Ω)	L_0 (H)	$f = 50$Hz. μ_I (A)	σ_I (A)	MSE_I (A^2)	C_{pk}	$f = 60$Hz. μ_I (A)	σ_I (A)	MSE_I (A^2)	C_{pk}
5	.02	12.458	.513	6.302	0.03	11.058	.462	1.333	1.04
7.1	.0225	9.984	.408	0.166	2.03	9.06	.371	1.053	1.39
7	.019	10.874	.446	0.962	1.22	9.988	.408	0.166	2.03
8	.017	10.4	.435	0.349	1.61	9.759	.401	0.219	1.88
9.43	**.01**	10.069	.452	**0.210**	1.79	9.854	.436	**0.211**	1.80

- Capability index

$$C_{pk} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\}$$

USL = 12.5 A
LSL = 7.50A

- Need compromise design for circuit to function 'equally' well under both frequencies → multiobjective optimization

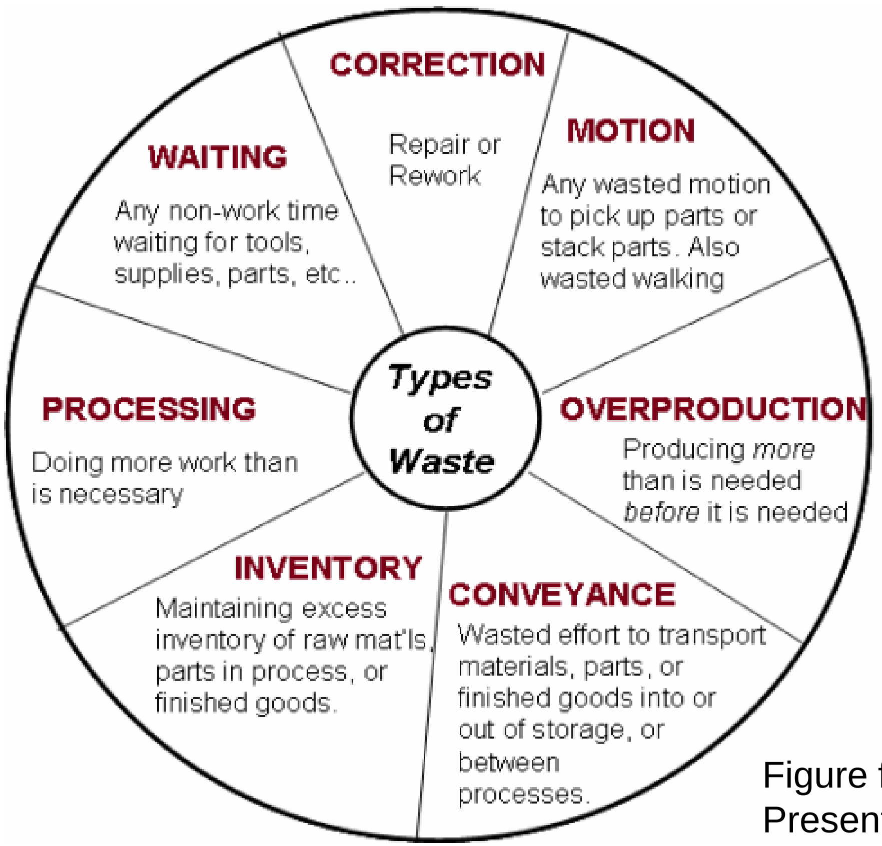
Quality Control and On-line Improvement

- Off-line design for quality: obtain best design based on the knowledge about the product and process *before* production
- Goal of on-line control: monitor manufacturing process for conformance to design specifications and tune parameters for further improvement
- Outline of topics

    1. Statistical Process Control (SPC) — general methodology
    2. Control Charts
    3. Process Capability Analysis (use of control charts for ...)
    4. Evolutionary Operation (EVOP) — on-line use of experiments

Why Speed?

- Speed ⇒ Optimized Process Flow ⇒ Lean
  - Lean Principles: Level Loading, Reduce Setups, Create Flows, Link suppliers, Time and waste Reduction,…

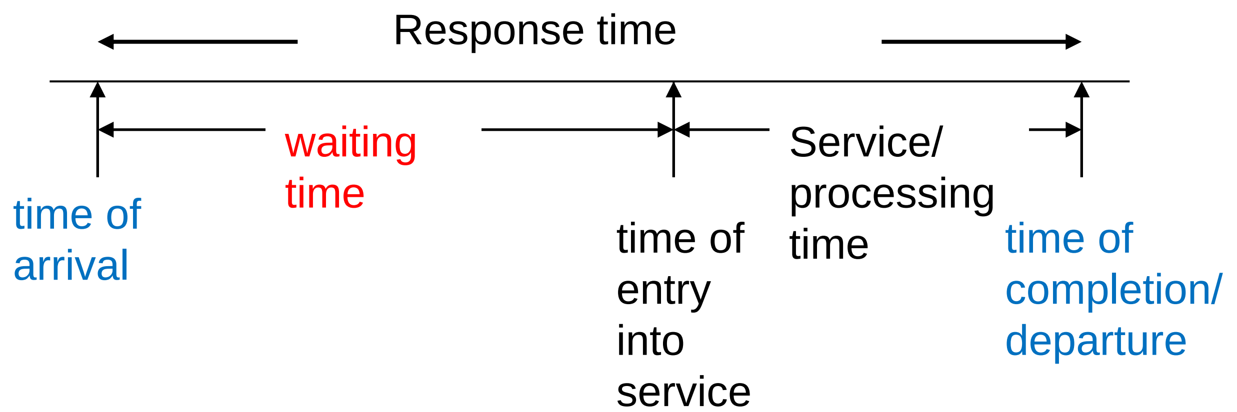


Lean identifies the sources of waste to reduce the Non-value added elements
⇒ Less work, less time, same result

Figure from: http://www.isssp.com/Media/LC04-Presentation-Slides/eckhardtezrabo.pdf

Performance Metrics - 1

1. Response time (Cycle Time):
   E [time of completion of a part - time of arrival of part]
   = Average time a part spends at each node (workstation)
   = Average waiting time + Average service time



$$R = W + \bar{t} = W + \frac{s}{\mu}$$

(Assuming a single server node)
Can also talk about system response time
= $\sum_i$ Response time at node *i*

2. Queue length (Work in Porcess (WIP))

   Average number of parts at each node (including the part in service) = Average number waiting + Average number in service $\Rightarrow Q = Q_W$ + Average number in service

Performance Metrics - 2

3. Throughput
   Average number of parts processed per unit time ⇒ a measure of productivity of the system

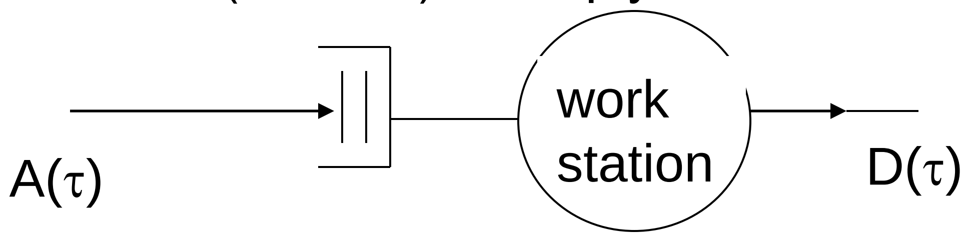
$$X = \frac{\text{Number of parts completed during } (t_o\,,\, t_f)}{\text{Observation interval } (t_f - t_o)} = \frac{C}{T}$$

   You can also talk of nodal and system throughputs.

4. Utilization of a node
   Fraction of the time (or the probability that) the node is busy

Little's Theorem - 1

Little's Theorem (formula) is simply an accounting identity.



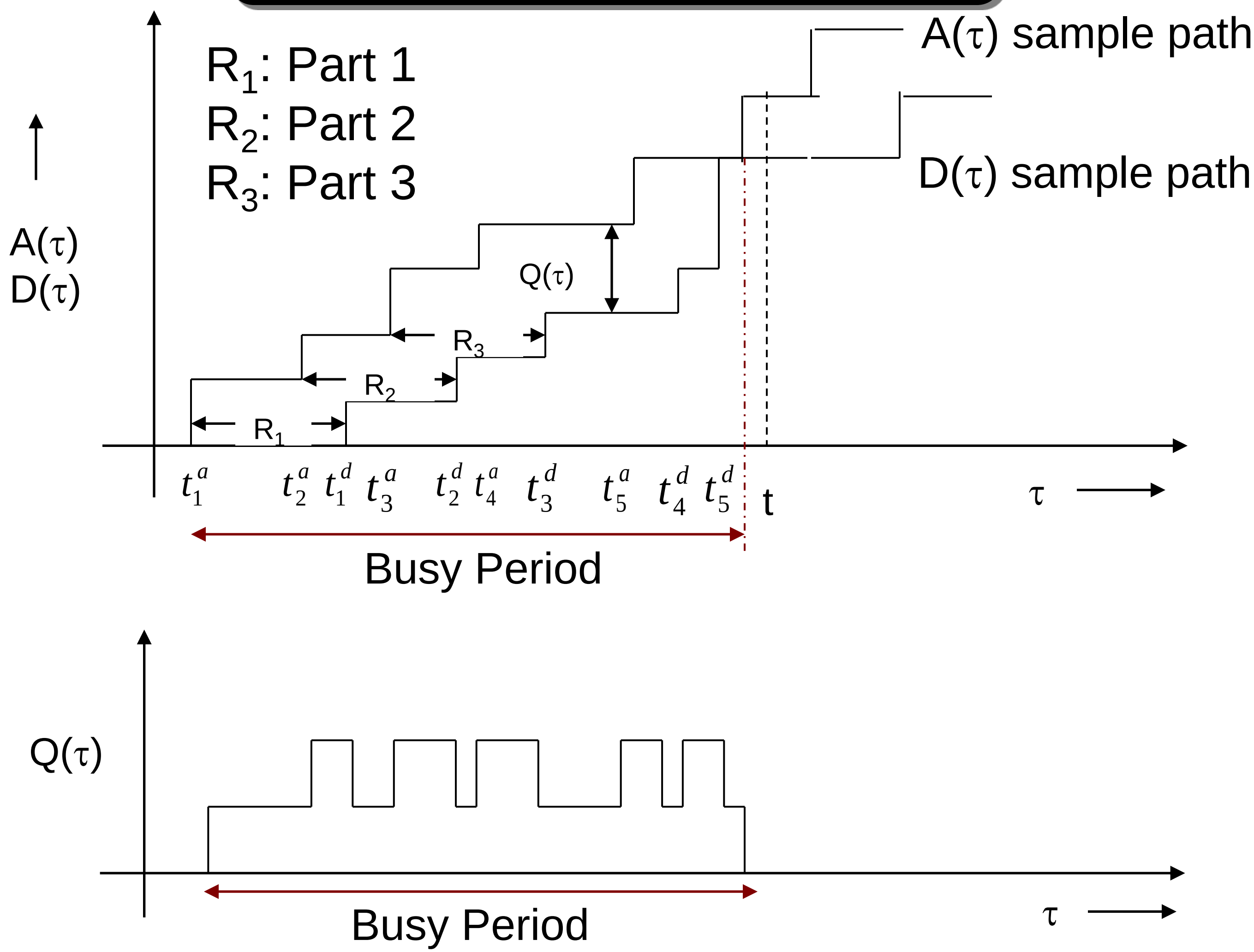
t_j^a Arrival time of j^{th} part

t_j^d Departure time of j^{th} part

$$Q(\tau) = A(\tau) - D(\tau)$$

Let us look at the sample paths of $A(\tau)$, $D(\tau)$ and $Q(\tau)$

Little's Theorem - 2



Little's Theorem - 3

Note that no assumption is made on the arrival or departure distributions. Also, no assumption is necessary on the scheduling discipline. Figure assumes FCFS, but is valid for any queuing system that reaches statistical equilibrium $\Rightarrow$ busy periods must be finite or $Q(\tau)$ is "ergodic."

Little's theorem relates:

- The average number of parts in the system (i.e., the "typical" # of parts either waiting in the queue or undergoing service), *Q* or WIP
- The average response time (cycle time) per part (i.e., the "typical" time a part spends waiting in the queue plus the service time), *R* in hours
- Part throughput in parts/hour. For open systems, we use the notation λ. For closed systems (CONWIP, knaban), we use the notation X.

Little's Law:

$$Q = \lambda R \text{ for open systems}$$

$$Q = X\, R \text{ for closed systems}$$

Proof of Little's Law - 3

Need to prove $Q = \lambda R$

We will show for FCFS only (LCFS and arbitrary service HW problem). In fact, it is valid for any scheduling discipline. Proof involves computing the area under the sample path curve in two ways:

One way: $\int_0^t Q(\tau)d\tau$

Second way: $\sum_{i=1}^{D(t)} R_i + \sum_{i=D(t)+1}^{A(t)} (t - t_i^a)$

Define $\overline{Q}(t) = \frac{1}{t}\int_0^t Q(\tau)d\tau$

= Time average of number of parts in the system in the interval [0,t]

Proof of Little's Law - 4

$$\bar{\lambda}(t) = \frac{A(t)}{t}$$ = Time average of part arrival rate in the interval [0,t]

$$\bar{R}_{D(t)}(t) = \frac{\sum_{i=1}^{D(t)} R_i + \sum_{i=D(t)+1}^{A(t)} (t - t_i^a)}{A(t)} =$$ Time average of response time

$$\bar{Q}(t) = \bar{\lambda}(t)\bar{R}_{D(t)}(t)$$

Taking $\lim_{t\to\infty}$

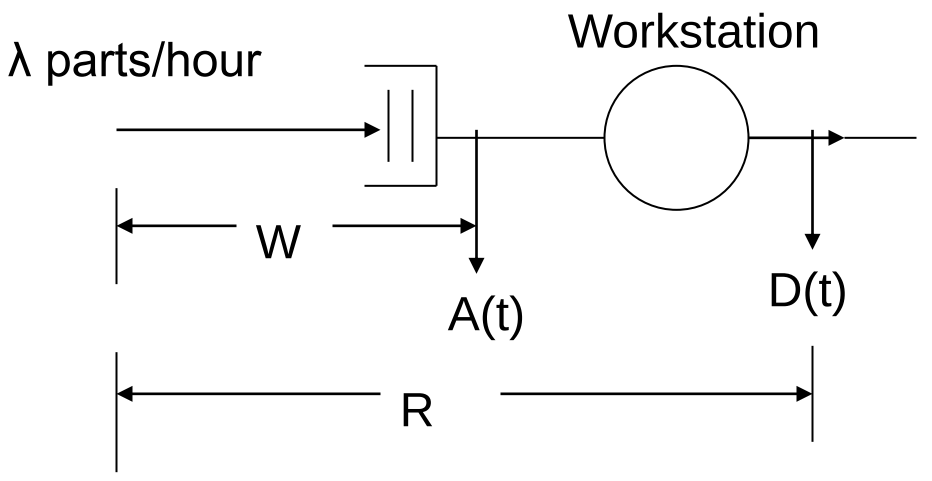
$Q = \lambda R$

$$\lim_{t\to\infty} \frac{A(t)}{t} = \lim_{t\to\infty} \frac{D(t)}{t}$$

Arrivals = Departures as $t \to \infty$

Applications of Little's Theorem -1

Example 1: Single server node (single workstation)



$$Q_W = \lambda W$$

$$U = \lambda \bar{t}$$

Utilization law is a special case of Little's formula!

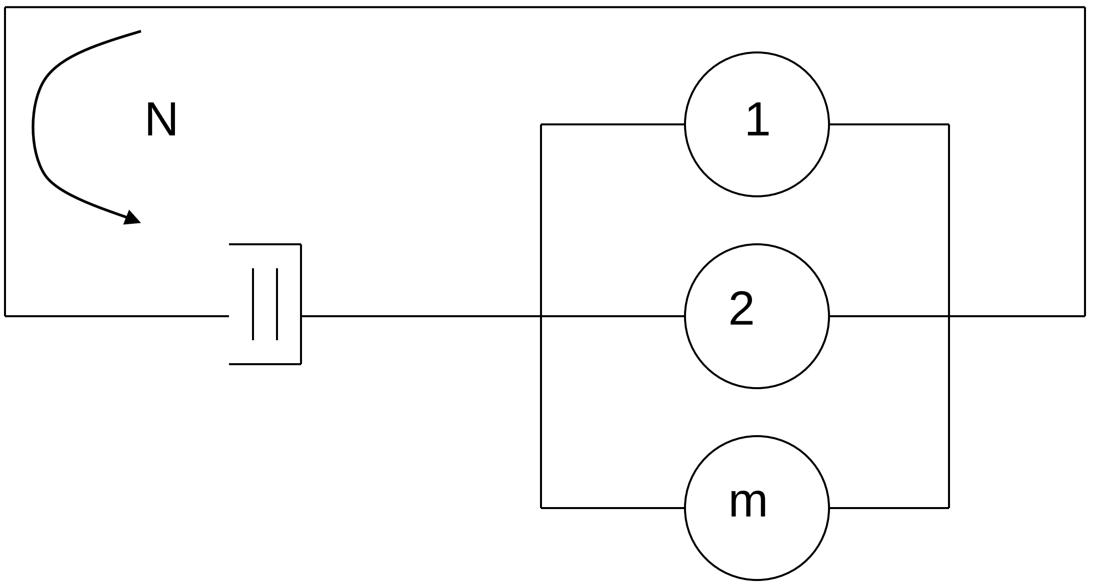
$$Q = \lambda R = Q_W + U$$

$$U \leq 1 \Rightarrow \lambda < \frac{1}{\bar{t}}$$ for stability

Throughput = $\boxed{\min(\lambda, \frac{1}{\bar{t}})}$

Applications of Little's Theorem -2

Example 2 : A closed system (CONWIP) with a multi-server node (Cell with multiple workstations)



N > m system is always full

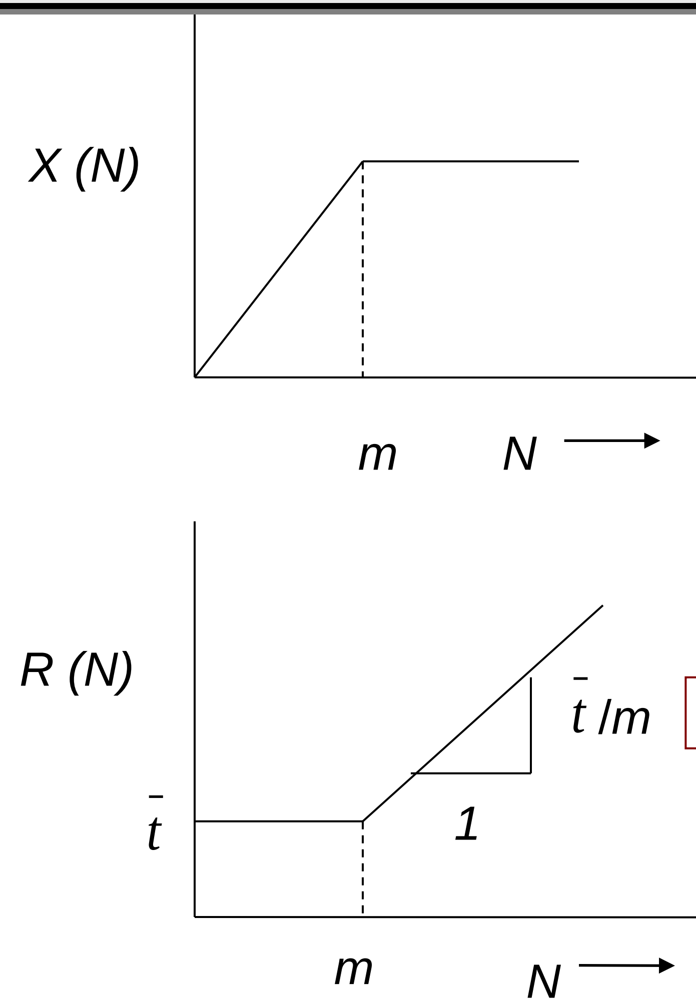
$$X(N)\bar{t} = \min(m, N)$$

$$X(N)R(N) = N$$

$so,$

$$R(N) = \frac{N}{\min(m, N)}\bar{t}$$

A Closed System with a Multi-server Node



$$X(N) = \frac{\min(m, N)}{\bar{t}}$$

Throughput versus *N*

$$R(N) = \frac{N}{\min(m, N)}\bar{t}$$

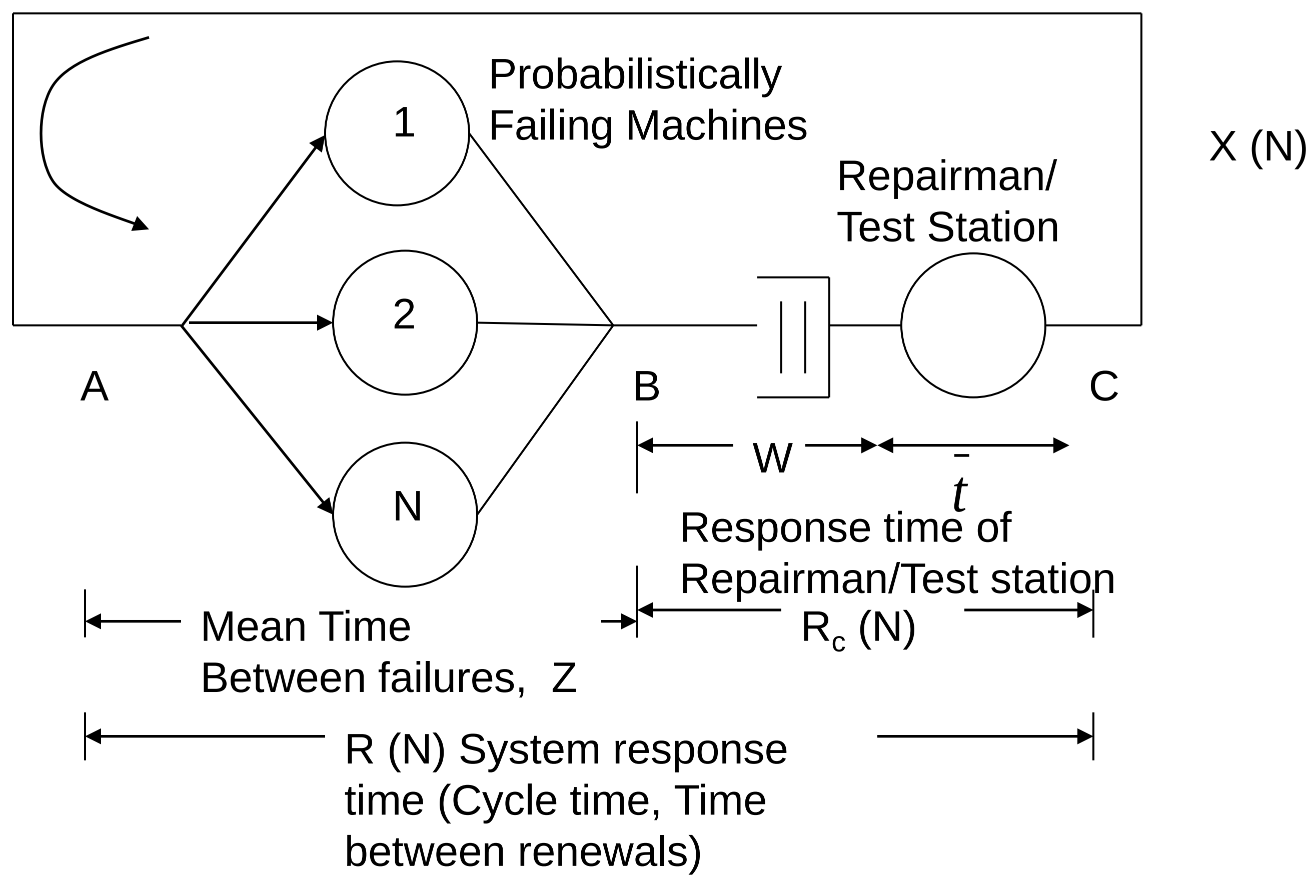
Response time (cycle time) versus *N*

$$m \to \infty \Rightarrow R(N) = \bar{t}$$

$$\Rightarrow \textit{no waiting}$$

Applications of Little's Theorem -3

Example 3: Machine Repairman Model or Machine interference model



Machine Repairman Model

Points A and C $X(N)R(N) = N \quad \Rightarrow X(N) = \dfrac{N}{R(N)}$

Also $R(N) = R_c(N) + Z$

We will obtain bounds on $R_c(N)$ via the so called Asymptotic Bounding Analysis (ABA).

Let us consider two extreme cases:

No waiting $\Rightarrow R_c(N) = \bar{t}$

Wait for (N-1) customers $\Rightarrow R_c(N) = (N-1)\bar{t} + \bar{t} = N\bar{t}$

Note: if multiple servers/repair men/ test stations: $R_c(N) = (N-m+1)\bar{t}$

So, $\bar{t} \le R_c(N) \le N\bar{t}$

$$\boxed{\bar{t} + Z \le R(N) \le N\bar{t} + Z}$$

$$\frac{1}{N\bar{t} + Z} \le \frac{1}{R(N)} \le \frac{1}{\bar{t} + Z}$$

Machine Repairman Model

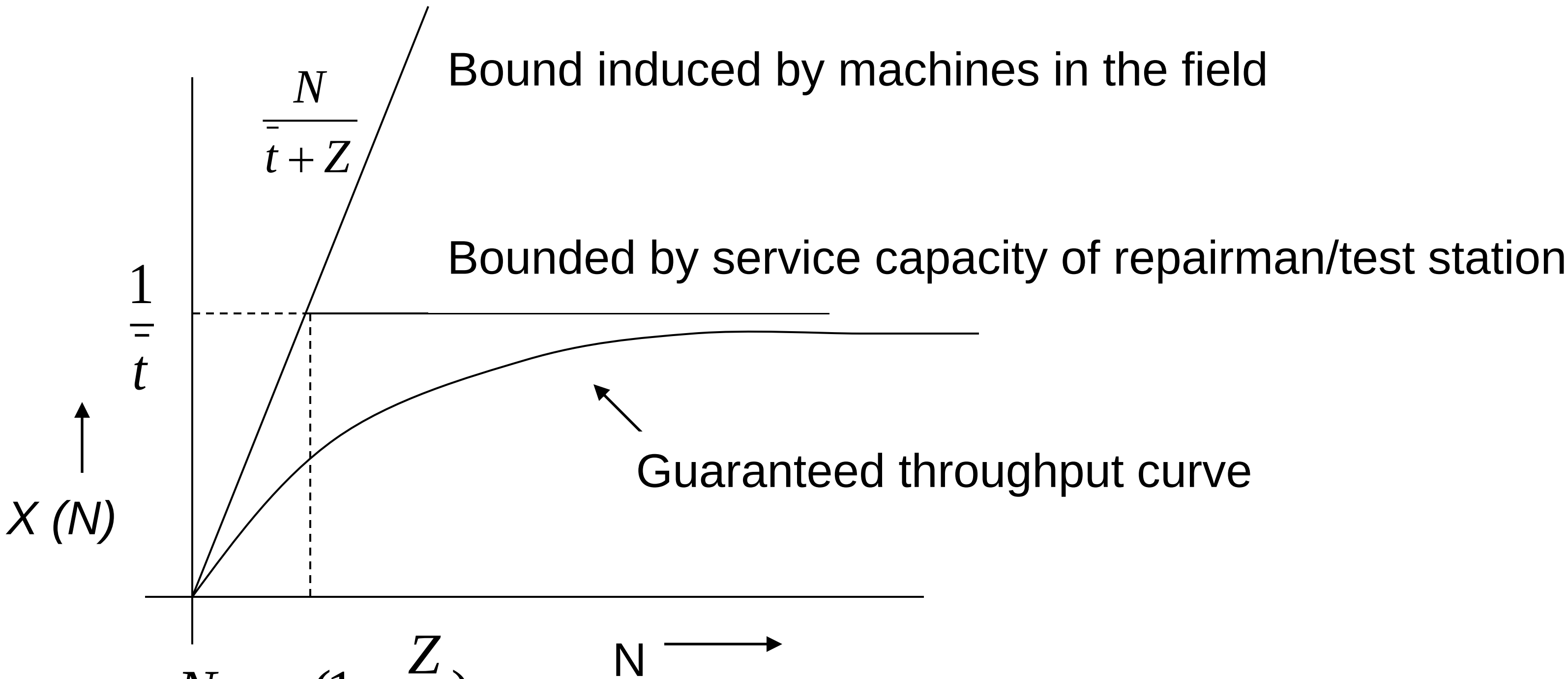
So, $$\frac{N}{N\bar{t}+Z} \le X(N) \le \frac{N}{\bar{t}+Z}$$

Also, since $X(N) \le \frac{1}{\bar{t}}$ (note for multi-server $X(N) \le \frac{m}{\bar{t}}$)

$$\frac{N}{N\bar{t}+Z} \le X(N) \le \min\left[\frac{N}{\bar{t}+Z}, \frac{1}{\bar{t}}\right]$$ ABA bounds

So $$\max\left(N\bar{t}, Z+\bar{t}\right) \le R(N) \le Z + N\bar{t}$$

Machine Repairman Model



$$N_{cr} = (1 + \frac{Z}{\bar{t}})$$

$$\frac{N_{cr}}{Z+\bar{t}} = \frac{1}{\bar{t}}$$

$$\Rightarrow N_{cr} = 1 + \frac{Z}{\bar{t}}$$

Machine Repairman Model

$N < 1 + \frac{Z}{\bar{t}} \Rightarrow$ Throughput limited by number of machines

$\Rightarrow$ repairman/test station is idle or most machines working!

$N > 1 + \frac{Z}{\bar{t}} \Rightarrow$ Throughput is limited by service capacity of repairman

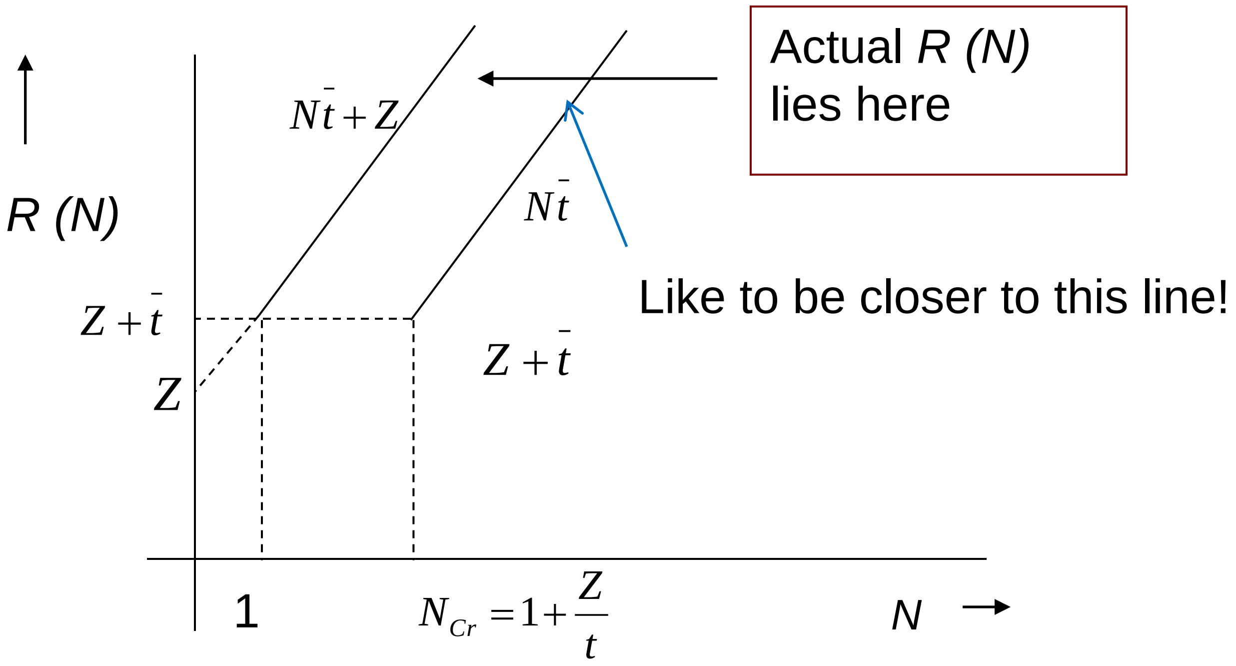
$\Rightarrow$ Repairman is saturated and linear increase in response time

$$1 + \frac{Z}{\bar{t}} = 1 + \frac{\text{Meantime to Failure (MTTF)}}{\text{Service time (Mean time to Repair (MTTR))}}$$

is called saturation point.

$\Rightarrow$ Suggests a method of selecting # of machines and # of repairmen.

Machine Repairman Model



Variability Matters in Lean Also

- Variability Measures:
  - Coefficient of variation (CV) of effective process times
  - Coefficient of variation of inter-arrival times

$$CV = \frac{\sigma}{\mu}$$

- Components of Process Variability
  - failures
  - setups
  - many others - deflate capacity *and* inflate variability
  - long infrequent disruptions worse than short frequent ones
- Consequences of Variability:
  - variability causes congestion (i.e., WIP/Cycle Time inflation)
  - variability propagates
  - variability and utilization interact
  - pooled variability less destructive than individual variability

Conclusion

- Quality : Key to Economic Success
  - Off-line Design for Quality and On-line Quality Control
  - Goal: Variability Reduction and Larger Process Capability
  - Six Sigma and Robust Design
- Lean ⇒ Increase speed and reduce waste
- Basic Factory Dynamics to Quantify Lean
  - Queuing Networks provide mathematical formalisms
  - Little's Theorem linking Work in Process (WIP, Queue length) with Cycle time (Response time) and Throughput
  - Variability makes a difference here as well!