Online Quality Control

Prof. Krishna R. Pattipati
Dept. of Electrical and Computer Engineering
University of Connecticut
Contact: krishna@engr.uconn.edu (860) 486-2890

ECE 6161

Modern Manufacturing System Engineering

Quality Control and Online Improvement

Offline design for quality: obtain best design based on the knowledge about the product and process *before* production

Goal of on-line control: monitor manufacturing process for conformance to design specifications and tune parameters for further improvement

Outline of topics

1. Statistical Process Control (SPC) – general methodology
2. Control Charts
3. Process Capability Analysis (use of control charts for ...)
4. Evolutionary Operation (EVOP) – on-line use of experiments
5. Quality and Manufacturing Operations

Process Improvement via SPC

- SPC provides information on
  - Statistical control of a process (Is the variation in process merely natural/unavoidable?)
  - Capability of process (How capable is the process in meeting specifications? How bad is the natural variability?)
- Recommended courses of action:

Is the process capable?

Is the process in control?	Yes	No
Yes	SPC	SPC and/or EVOP Experimental design Change Process
No	SPC	SPC Experimental design Investigate specifications Change process

The Control Chart

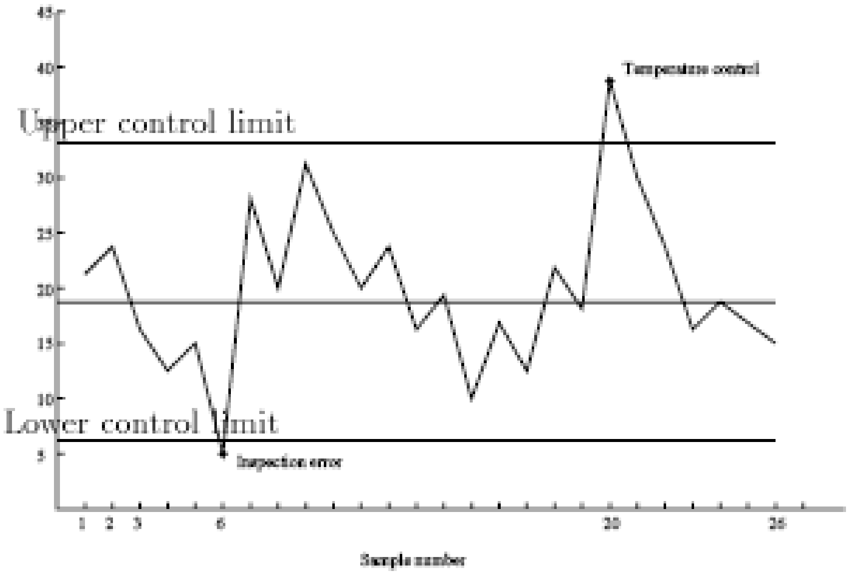
Used to

Detect out-of-control change in a process (primary goal)

Estimate process parameters – determine process capability

Obtain information for improving process and reducing variability

A typical control chart



The Control Chart: General Model

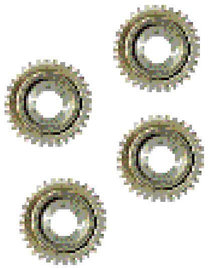
- Called Shewhart Control Charts [Dr. Walter A. Shewhart (1930's)]
- Plot w: a sample statistic that measures a quality characteristic
  - μ_w: mean of w
  - σ_w: standard deviation of w

$$\text{UCL} = \mu_w + k\sigma_w$$
$$\text{Center line} = \mu_w$$
$$\text{LCL} = \mu_w - k\sigma_w$$

- k: “distance” of control limits from center line in units of standard deviation; typically $k = 3$ (3σ control limits→99.73% confidence for Normal distribution)
- Control chart essentially a repeated test of **null hypothesis** that the process is in control (hypothesis that w is distributed with mean and standard deviation corresponding to in-control state)

Computing Control Chart Parameters

- Problem: control diameter of hole in steel castings
  - desired nominal diameter of $\mu = 10$ mm
  - observations have shown $\sigma = 0.025$ mm



- Process: every 2 hours a casting is randomly selected, so

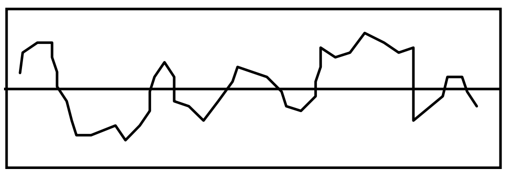
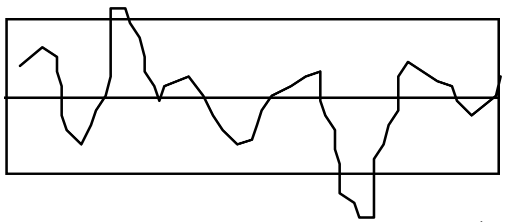
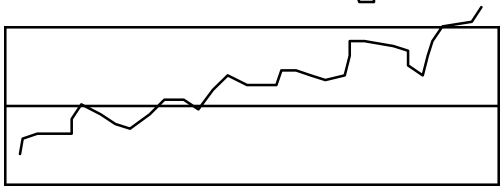
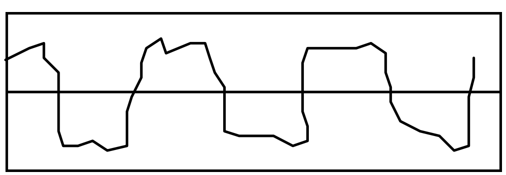
$$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 0.025 / \sqrt{1} = 0.025$$

$$\text{LCL} = \mu - 3\sigma_{\bar{x}} = 10 - 3(0.025) = 9.925$$

$$\text{UCL} = \mu + 3\sigma_{\bar{x}} = 10 + 3(0.025) = 10.075$$

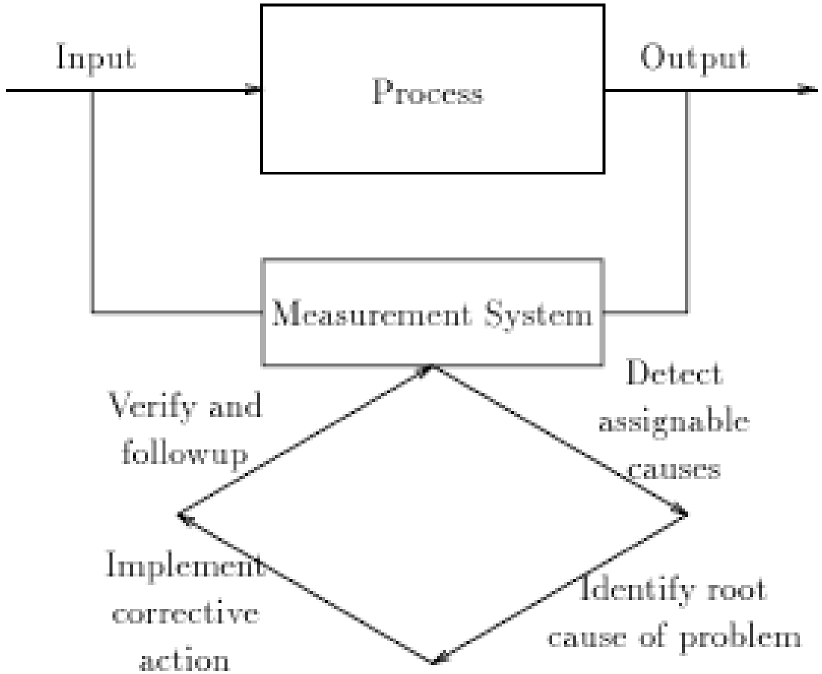
Note: variability would be reduced by taking n>1, due to pooling.

Control Chart Patterns

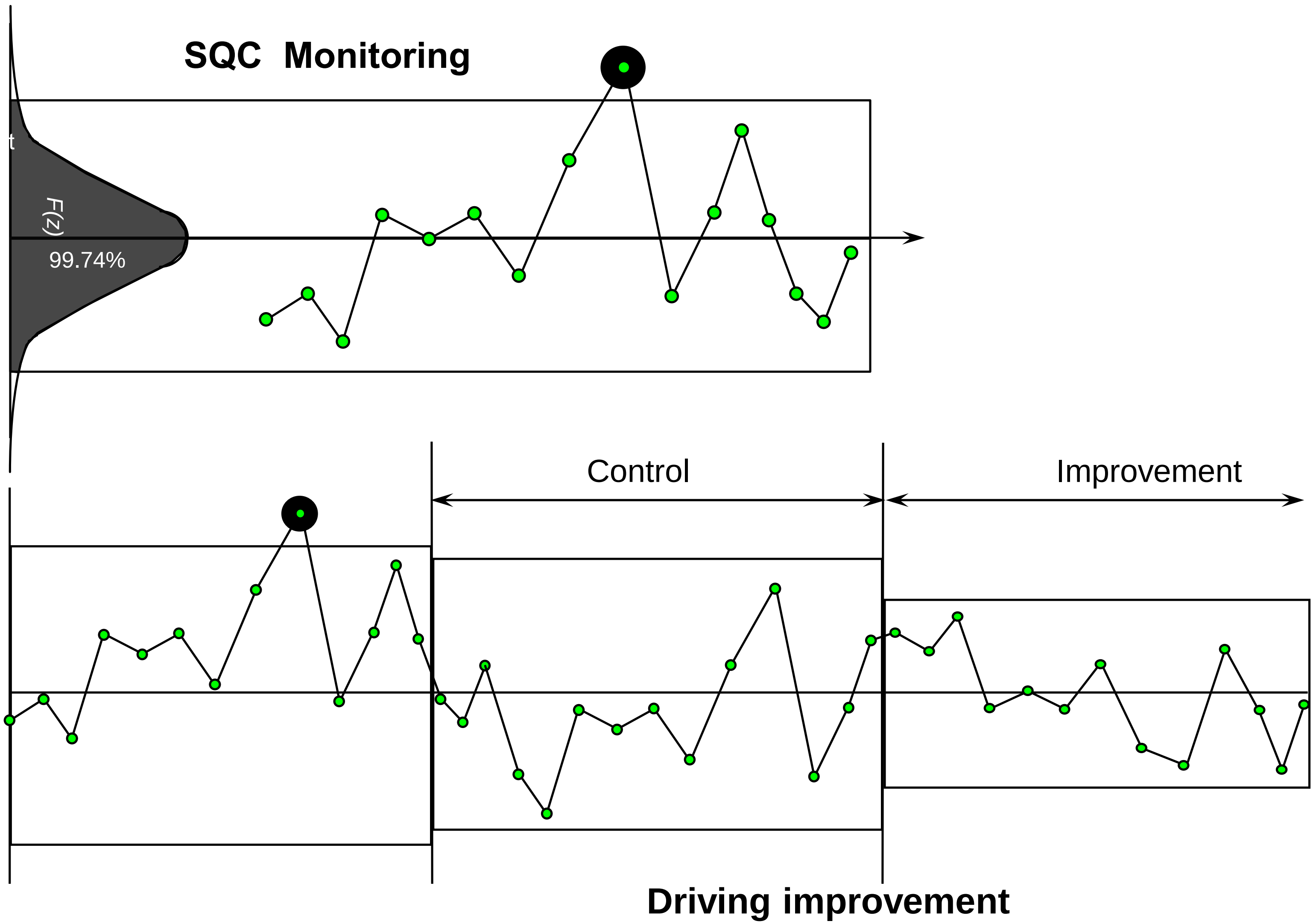
Pattern	Description	Possible Causes
	Normal	Random Variation
	Lack of Stability	Assignable (or special) causes (e.g., tool, material, operator, overcontrol
	Cumulative trend	Tool Wear
	Cyclical	Different work shifts, voltage fluctuations, seasonal effects

Improvement via Control Charts

- Most processes do not operate in statistical control => routine use of control chart can identify assignable causes
- Control chart can only detect assignable causes: management, operator, and engineering action necessary to eliminate the causes => process *improved* by reducing variability



Continuous Improvement



Utility of Control Charts

A technique for improving productivity – reduce scrap/rework

Defect prevention–"do it right the first time"

Prevent unnecessary adjustments in response to background noise (do not over-react to possibly natural variation)

Provide diagnostic information

Provide information about process capability – useful for product and process designers (how much really is the natural variability?)

Example Uses of Control Charts

Product Quality
- Dimensions and other physical attributes
- Fraction nonconforming
- Range of attributes (for monitoring variability)

Times
- Process times
- Repair times

Other Non-Quality Applications
- Tracking throughput
- Due date quoting

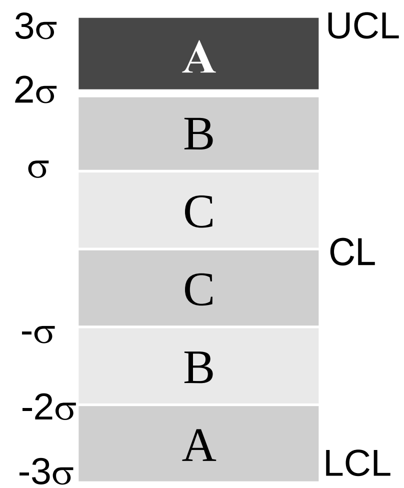
Control Charts: Design Issues

- Choice of control limits: based on risk (probability) of making an error
  - Type I error: point falls outside control limits even when no assignable cause present (a.k.a. false alarm)
  - Type II error: point falls inside control limits when process actually out of control (a.k.a. missed detection)
- Warning limits: 2-sigma limits in addition to 3-sigma control-limits - if sample-point falls outside warning limits but inside control limits take additional data to investigate state of control of process
- Allocation of sampling effort: sample size and sampling frequency
  - Larger sample size => enables detection of small shifts in process
  - Frequent sampling => early detection of out-of-control state
- Current practice: take smaller, more frequent samples
- Can also base decision on *average run length* (ARL)

AT&T Rules for Control Charts

Investigate if

- 2 out of 3 points in a row in zones A and above
- 4 out of 5 in a row in B or above
- 8 consecutive in C or beyond
- 1 point beyond A
- 6 points in a row steadily increasing
- 6 points in a row steadily decreasing
- 14 points in a row alternating up and down



Control Charts: Design Issues

- ARL (Average Run Length) of control-chart: average number of points plotted before out-of-control situation is indicated
  - Shewhart control-charts (only the most recent sample statistic used to test in-control hypothesis):

$$\text{ARL} = \frac{1}{p}$$

p : probability that any point exceeds control limits

  - Example: 3-σ control limits => p = 0.0027 when process in control

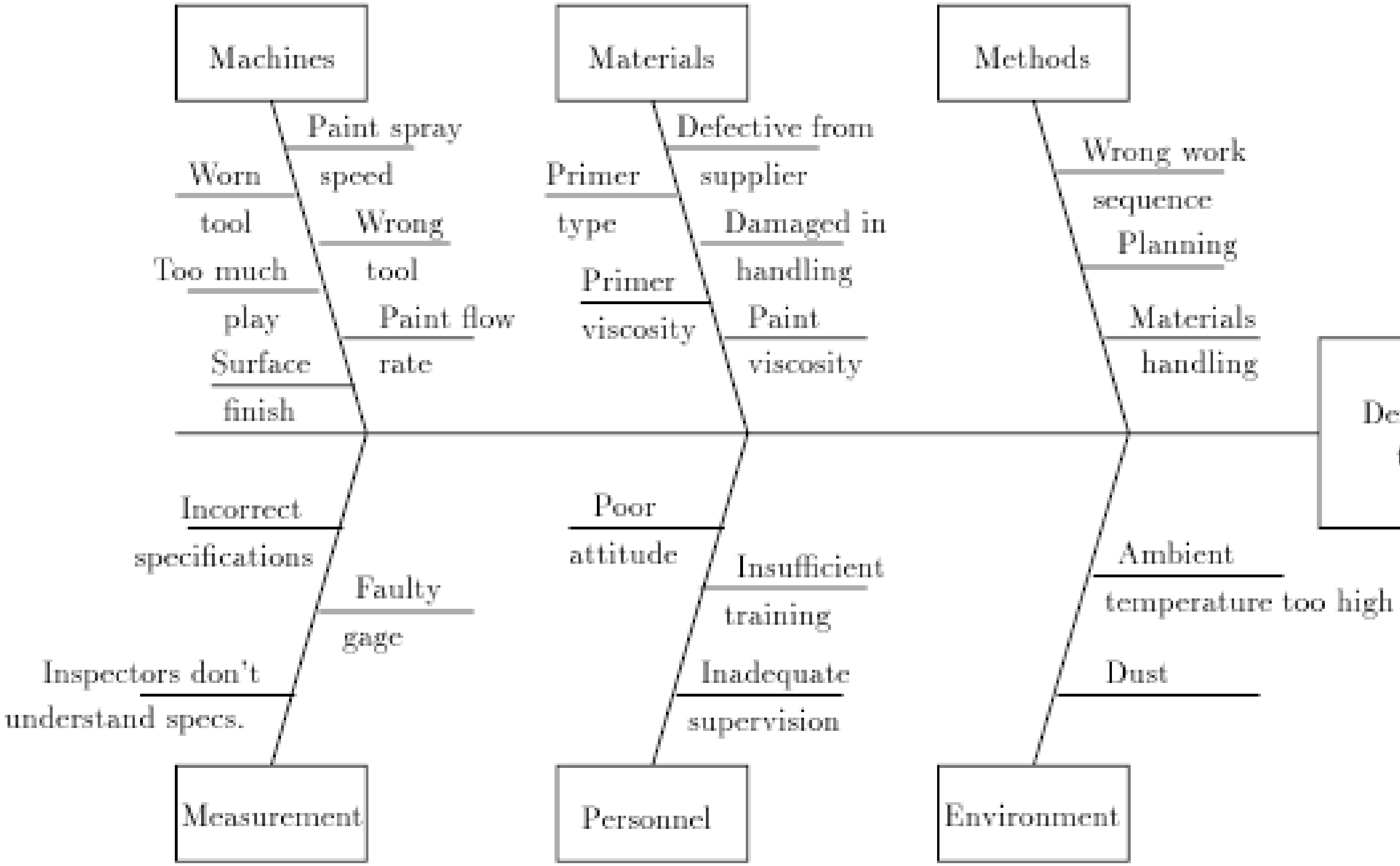
$$\text{ARL} = \frac{1}{0.0027} = 370$$

$\Rightarrow$ 370 samples plotted before false-alarm

  - Mean shifts from center-line => *p* increases => ARL reduces (need fewer points to detect actual out-of-control)
- *Rational subgroups:* samples (subgroups) should be chosen so that if assignable cause(s) present, chance for differences between subgroups is maximized and chances for differences within subgroups are minimized

Cause and Effect Diagrams

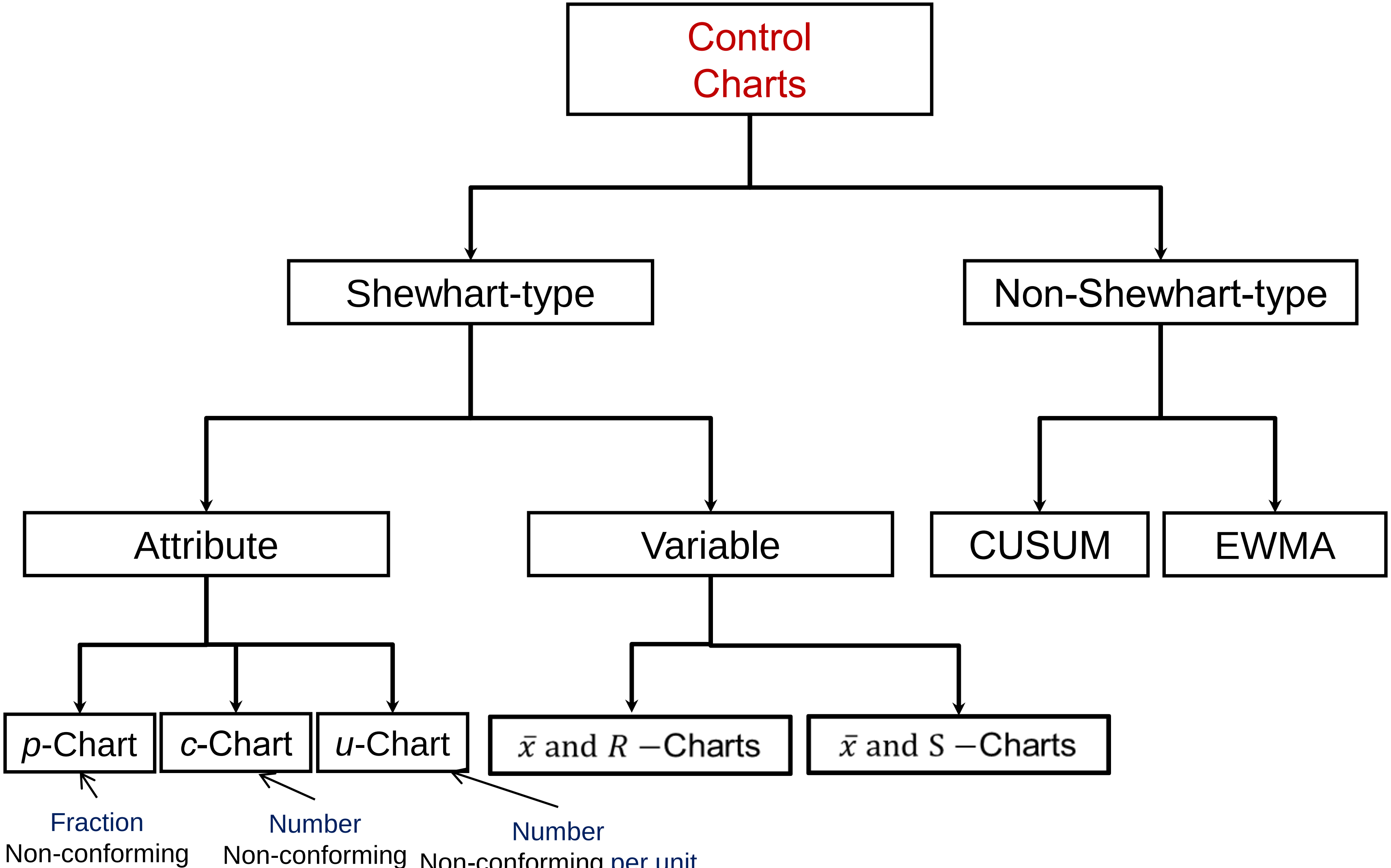
- Cause and Effect Diagram: formal tool useful in unlayering potential causes of an undesirable effect (Ishikawa/Fishbone/Herringbone diagrams)



Constructing a CE Diagram

- Start with a symptom: a condition where evidence of a problem is manifested ("observed effect")
- Ask: What are the major stimuli ("root causes") behind the observed effect?
- Process of constructing a CE diagram:
  - Start with a symptom and draw the basic shell ("fishbone")
  - Identify the major causes
  - Brainstorm for all possible causes
  - Circle the root causes, then prioritize them
  - Verify the selected major causes with further data collection

Control Charts: Classification



Control Charts for Attributes

Attributes: quality characteristics that cannot be represented numerically

Product declared *conforming/nonconforming* to the specifications of an attribute-type quality characteristic

Three widely used control charts for attributes

- p chart: plot fraction of nonconforming products
- c chart: plot number of nonconformities or defects
- u chart: plot number of nonconformities per unit

p-Chart: Control Chart for Fraction Nonconforming

- Fraction nonconforming = $\dfrac{\text{Number of nonconforming items in a population}}{\text{Total number of items in the population}}$

- Statistical principle: based on the *binomial distribution*

- *p:* probability that any unit will not conform to specifications

- *X:* number of units of product that are nonconforming in a random sample of *n* units

- Probability that $X = x$ units out of n are nonconforming

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\text{Mean of } x: \mu_X = np$$

$$\text{Variance of } x: \sigma_X^2 = np(1-p)$$

p-Chart (cont'd)

- Statistic plotted on a *p*-chart: $\hat{p} = \frac{X}{n}$

$$\text{Mean of } \hat{p} : \mu_{\hat{p}} = p(\text{unbiased})$$

$$\text{Variance of } \hat{p} : \sigma_{\hat{p}}^2 = \frac{p(1-p)}{n}$$

- Center line and 3-σ control-limits of a *p*-chart

$$\text{UCL:} p + 3\sqrt{\frac{p(1-p)}{n}}$$

$$\text{Center line:} p$$

$$\text{LCL: } p - 3\sqrt{\frac{p(1-p)}{n}}$$

- *p* is not known => estimate from *m* preliminary samples (typically 20-25) each of size *n*
  - If D_i nonconforming units in i^{th} sample, $\hat{p}_i = \frac{D_i}{n}, i = 1, 2, \ldots.., m$
  - Estimate p by $\bar{p}$:

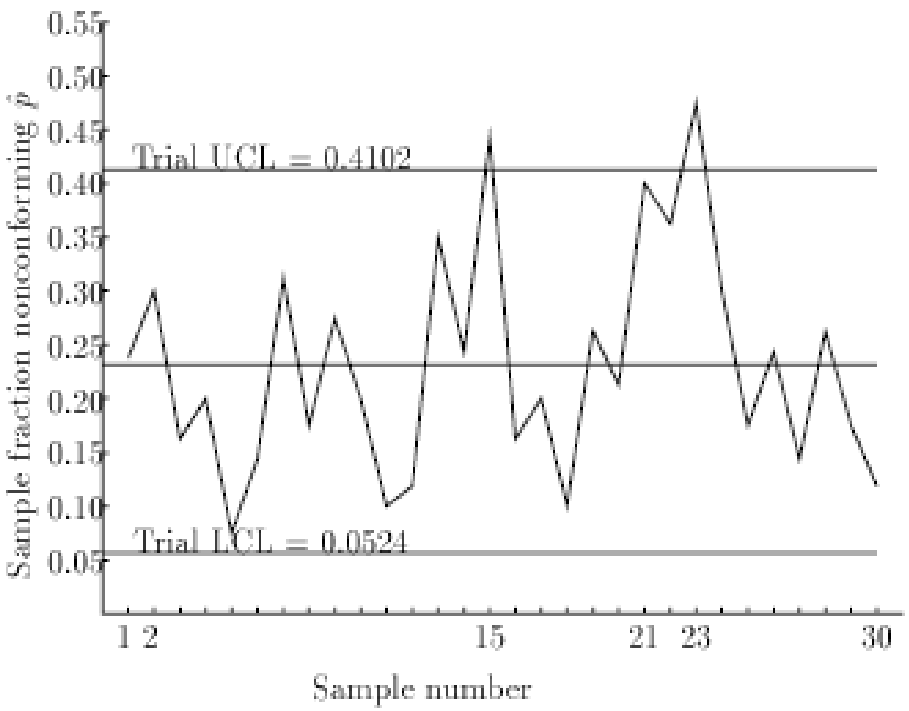
$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn}$$

p-Chart Example

$$\bar{p} = 347/(30)(50) = 0.2313$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2313 + 0.1789 = 0.4102$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.2313 - 0.1789 = 0.0524$$



Data for trial control limits, sample size $n = 50$

Sample Number	Number of Nonconforming Units	Sample Fraction Nonconforming
1	12	0.24
2	15	0.30
3	8	0.16
4	10	0.20
5	4	0.08
6	7	0.14
7	16	0.32
8	9	0.18
9	14	0.28
10	10	0.20
11	5	0.10
12	6	0.12
13	17	0.34
14	12	0.24
15	22	0.44
16	8	0.16
17	10	0.20
18	5	0.10
19	13	0.26
20	11	0.22
21	20	0.40
22	18	0.36
23	24	0.48
24	15	0.30
25	9	0.18
26	12	0.24
27	7	0.14
28	13	0.26
29	9	0.18
30	6	0.12
	347	$\bar{p} = 0.2313$

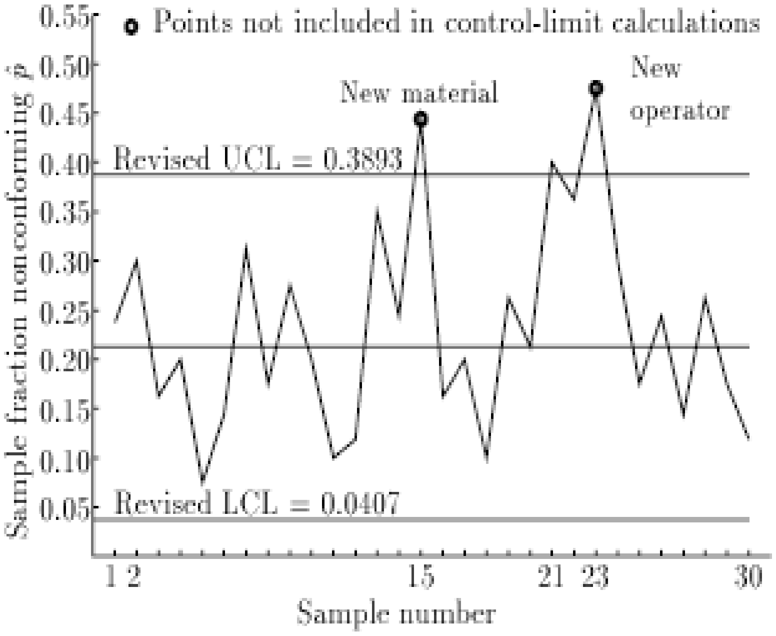
p-Chart Example (cont'd)

- Samples 15 and 23 outside control limits; any assignable causes?
  - Sample 15: new batch of raw material introduced (possibly caused irregular production performance)
  - Sample 23: Inexperienced operator temporarily assigned
- Eliminate samples 15 and 23 and calculate new control limits

$$\bar{p} = 301/(28)(50) = 0.2150$$
$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.3893$$
$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0407$$

Sample 21 now exceeds UCL — retain if no assignable cause found

New control limits adopted for subsequent monitoring



c-Chart: Control Chart for Nonconformities (Defects in a Unit)

Several defects/nonconformities possible in a single product

- Number of broken rivets in an aircraft wing
- Number of defective welds in 100m of oil pipeline

Assumption: occurrence of defects in samples of constant size (*inspection units*) modeled by *Poisson distribution*

x: number of nonconformities in an inspection unit

Probability of *x* nonconformities

$$p(x) = \frac{e^{-c}c^{x}}{x!}, x = 0,1,2,\ldots$$

$c > 0$: parameter of the Poisson distribution

Mean of x = Variance of $x = c$

c-Chart (cont'd)

- Statistic plotted on a c-chart: number of defects *x*
- Center line and 3-σ control-limits of a *c*-chart

$$\text{UCL} = c + 3\sqrt{c}$$

$$\text{Center line} = c$$

$$\text{LCL} = c - 3\sqrt{c}$$

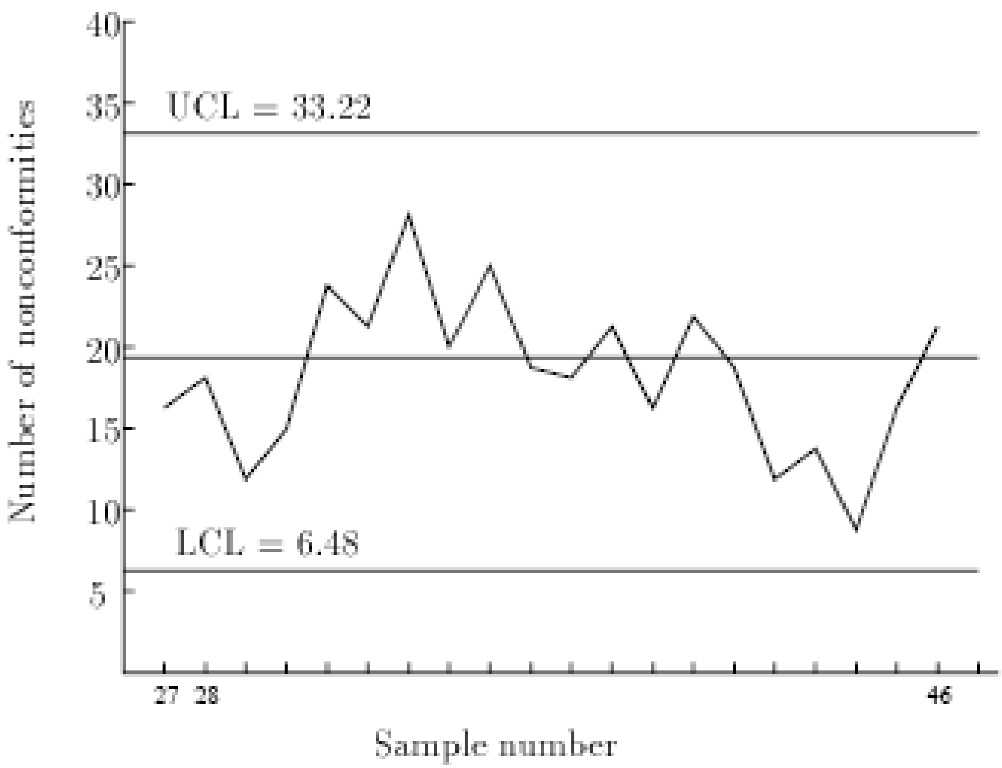
- *c* not known => use estimate $\bar{c}$ obtained from preliminary samples

c-Chart Example

$$\bar{c} = 516/26 = 19.85$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 19.85 + 3\sqrt{19.85} = 33.22$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 19.85 - 3\sqrt{19.85} = 6.48$$



Number of defects in samples of 100 printed circuit boards

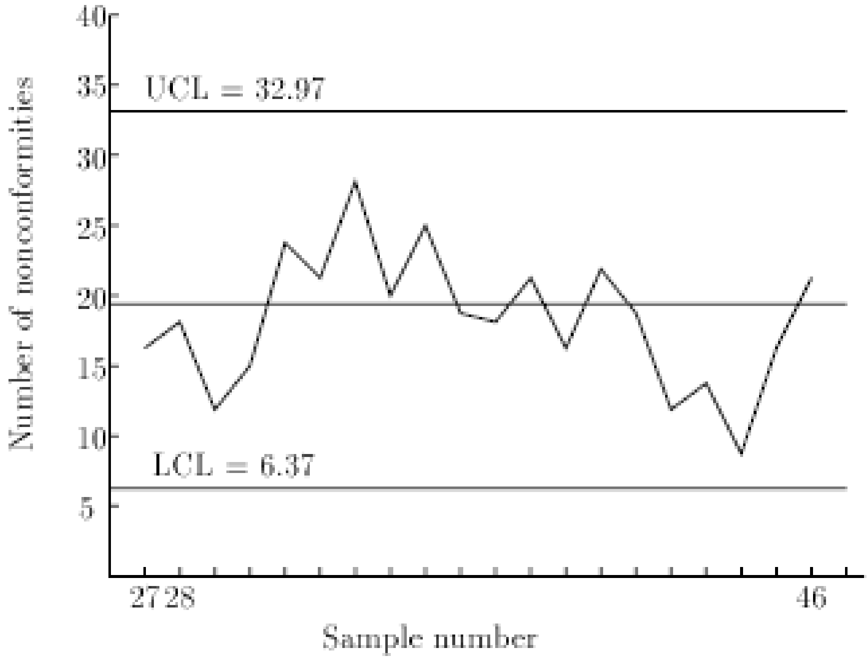
Sample Number	Number of Nonconformities
1	21
2	24
3	16
4	12
5	15
6	5
7	28
8	20
9	31
10	25
11	20
12	24
13	16
14	19
15	10
16	17
17	13
18	22
19	18
20	39
21	30
22	24
23	16
24	19
25	17
26	15
	516

c-Chart Example (cont'd)

- Assignable causes found for samples 6 and 20 → revise control limits

Use revised limits as standard for next period

$$\begin{aligned} \bar{c} &= 472/24 = 19.67 \\ \text{UCL} &= \bar{c} + 3\sqrt{\bar{c}} = 19.67 + 3\sqrt{19.67} = 32.97 \\ \text{LCL} &= \bar{c} - 3\sqrt{\bar{c}} = 19.67 - 3\sqrt{19.67} = 6.37 \end{aligned}$$



Additional defect data for printed-circuit-boards example

Sample Number	Number of Nonconformities
27	16
28	18
29	12
30	15
31	24
32	21
33	28
34	20
35	25
36	19
37	18
38	21
39	16
40	22
41	19
42	12
43	14
44	9
45	16
46	21

u-Chart: Control Chart for Average Nonconformities per Unit

Use n inspection units

c total nonconformities in n inspection units

Average nonconformities per inspection unit

$$u = \frac{c}{n}$$

c is Poisson random variable =>

$$\text{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

$$\text{Center line} = \bar{u}$$

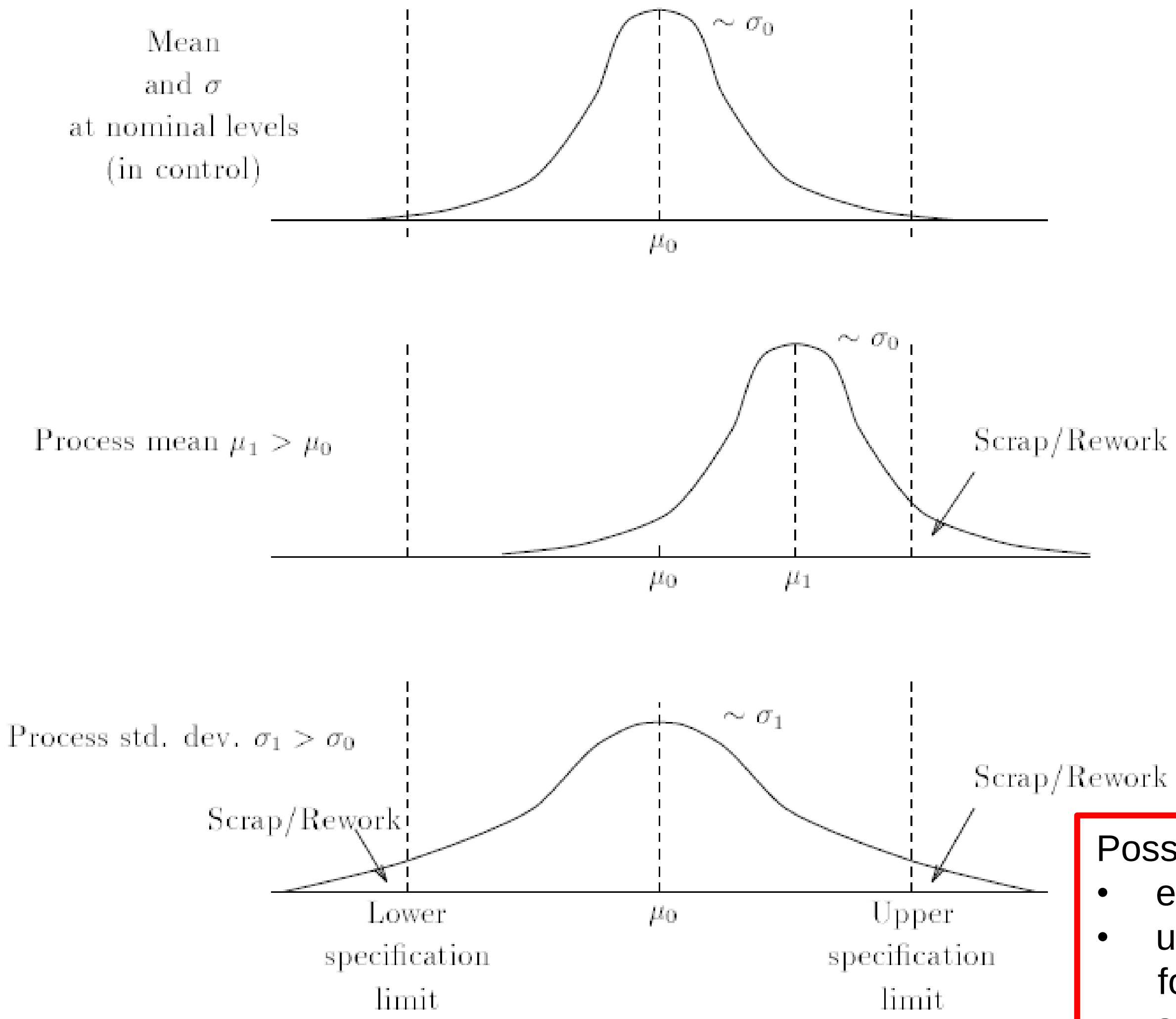
$$\text{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

$\bar{u} \rightarrow$ estimated average nonconformities per unit from preliminary data

Control Charts for Variables

- Variable: a single measurable (quantitative) quality characteristic, e.g., a dimension, weight, or volume
- Control charts for variables provide more information about process performance than *attribute* control charts
- Need to control both mean and variability of the quality characteristic
  - Control chart for mean of variable: $\bar{x}$-chart
  - Control chart for variability: two options
    - *S*-chart (for standard deviation)
    - *R*-chart (for range) → more frequently used

Need to Control Both Mean and Variability



Possible Cures of rework:

- eliminate rework
- use non-bottleneck for reworking
- shorten rework loop

$\bar{x}$- and R-Charts

Assume quality characteristic is normally distributed as $N(\mu, \sigma)$

Sample of size n of the quality characteristic considered: $x_1, x_2, \ldots, x_n$

Statistic for $\bar{x}$ -chart: sample average

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

$\bar{x}$ is distributed as $N(\mu, \sigma/\sqrt{n})$

3-σ control limits of $\bar{x}$-chart:

$$\text{UCL} = \mu + 3\frac{\sigma}{\sqrt{n}}$$

$$\text{Center line} = \mu$$

$$\text{LCL} = \mu - 3\frac{\sigma}{\sqrt{n}}$$

μ and σ not known; estimated from preliminary samples

$\bar{x}$- and R-Charts : Estimation of Control Limits

- μ and σ not known; estimated from m preliminary samples
  - $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m$, : average of each sample of size n
  - – $\bar{\bar{x}}$ used as center line Estimate of $\mu = \bar{\bar{x}} = \dfrac{\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_m}{m}$
- Usual (quadratic) method of estimating σ : from sample variance S^2

$$\hat{\sigma}_l = \sqrt{S_l^2} = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i^l - \bar{x}_l)^2}; \hat{\sigma} = \sqrt{\frac{1}{m}\left[\sum_{l=1}^{m}\hat{\sigma}_l^2 + \left(\bar{x}_l - \bar{\bar{x}}\right)^2\right]}$$

- Range method to estimate σ : almost as good as quadratic estimator for small sample sizes (n < 10); relative efficiency deteriorates as n increases
  - Small samples: typically 4, 5, or 6 due to rational subgrouping, high cost of sampling and inspection associated with variable measurements

Range Method and *R*-Charts

- Range method to estimate σ
  - Range of sample: difference between the largest and smallest observations $R = x_{\max} - x_{\min}$
  - Define relative range $W = R/\sigma$
  - d_2: mean of W - tabulated values available ($d_2 \sim 1.1$-3.9 for $n \sim 2$- 25)
  - Estimate σ by $\hat{\sigma} = \bar{R}/d_2$, $\bar{R} = (R_1 + R_2 + \ldots + R_m) / m$
- R-chart: plot range values from successive samples to control variability
  - Standard deviation of R, σ_R : $\sigma_R = d_3\sigma$
  - d_3: standard deviation of W-tables of values available ($d_3 \sim 0.7$-0.85)
  - Estimate of σ_R: $\hat{\sigma}_R = d_3 \frac{\bar{R}}{d_2}$
  - Control limits

$$\text{UCL} = \bar{R} + 3d_3\frac{\bar{R}}{d_2} \quad \text{Center line} = \bar{R} \quad \text{LCL} = \bar{R} - 3d_3\frac{\bar{R}}{d_2}$$

n	d_2	d_3
20	3.735	0.729
25	3.931	0.708

Control Charts for Individual Measurements

- What if $n = 1$? (sample for inspection is an individual unit), e.g.,
  - every unit is analyzed (e.g., use of automated inspection and measurement)
  - slow rate of production - cannot allow sample sizes of $n > 1$ to accumulate
  - measurements made on a batch differ very little - treated as one measurement (e.g., thickness at various locations of a roll of paper)
- Options
  - Control chart for individual units
  - Cumulative sum (CUSUM) or exponentially-weighted moving-average (EWMA) control charts - for detecting small shifts in process (discussed later)
- Control chart for individual units: in manner of $\bar{x}$- and R-charts
  - Plot individual measurements, and
  - Plot variability measure estimated from moving range of two successive observations

Individuals Control Chart Example

- Quality characteristic: viscosity of primer paint for aircrafts
- Control limits for MR-chart (using n = 2 for moving range)

$$\text{UCL}=\overline{MR}(1+3\frac{d_3}{d_2}) = 0.48(3.267) = 1.57$$

$$\text{Center line}=\overline{MR} = 0.48$$

$$\text{LCL}=\overline{MR}(1\text{-}3\frac{d_3}{d_2}) = 0.48(0) = 0$$

$d_3 = 0.8525$
$d_2 = 1.1280$

$$[\text{For } n = 2, d_2 = 1.128, d_3 = 0.853,$$

$$1 + 3\frac{0.853}{1.128} = 1 + 2.267 = 3.267]$$

- Control limits for individual-measurement chart

$$\text{UCL}=\bar{x} + 3\frac{\overline{MR}}{d_2} = 33.52 + 3\frac{0.48}{1.128} = 34.80$$

$$\text{Center line}=\bar{x} = 33.52$$

$$\text{LCL}=\bar{x} - 3\frac{\overline{MR}}{d_2} = 33.52 - 3\frac{0.48}{1.128} = 32.24$$

Viscosity of aircraft primer paint

Batch Number	Viscosity x	Moving Range MR
1	33.75	
2	33.05	0.70
3	34.00	0.95
4	33.81	0.19
5	33.46	0.35
6	34.02	0.56
7	33.68	0.34
8	33.27	0.41
9	33.49	0.22
10	33.20	0.29
11	33.62	0.42
12	33.00	0.62
13	33.54	0.54
14	33.12	0.42
15	33.84	0.72
	$\bar{x} = 33.52$	$\overline{MR} = 0.48$

Individuals Control Chart Example (cont'd)

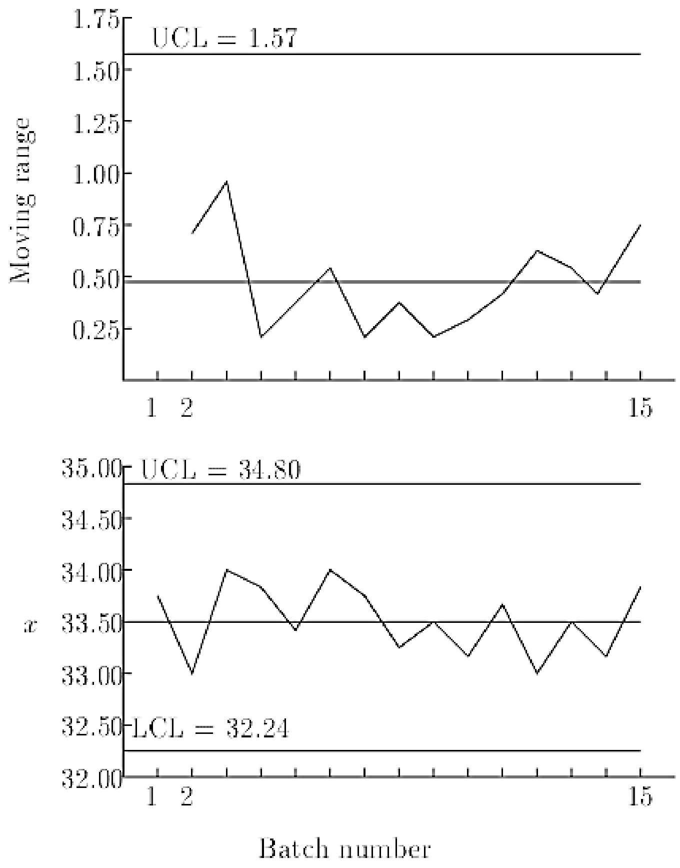
Control charts for moving range and individual observations on viscosity

Process is in control

Note on interpretation:

MR-chart is correlated

x measurements are assumed uncorrelated $\Rightarrow$any pattern in *x*-chart must be investigated



The Cumulative-Sum Control Chart

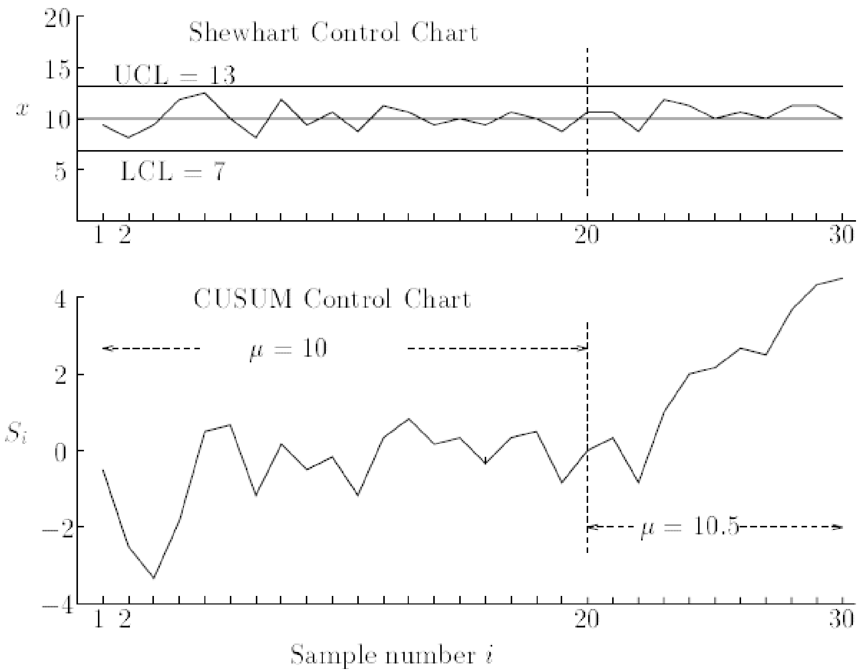
- $\bar{x}_j$: average of j^{th} sample (or x_j if sample size $n = 1$)
- μ_0: target for process mean
- CUSUM chart: plot cumulative sum S_i against sample number i

$$S_i = \sum_{j=1}^{i} \left(\overline{x}_j - \mu_0 \right) = S_{i-1} + \left(\overline{x}_i - \mu_0 \right)$$

  - combine information from several samples - effective for detecting small shifts
  - good for $n = 1$
- Trends in CUSUM chart
  - If process is in control at target value μ_0, S_i should fluctuate about zero (random walk with mean zero)
  - If process mean $\mu_1 > \mu_0$, upward drift in S_i
  - If process mean $\mu_1 < \mu_0$, downward drift in S_i
- Control limits: V-mask

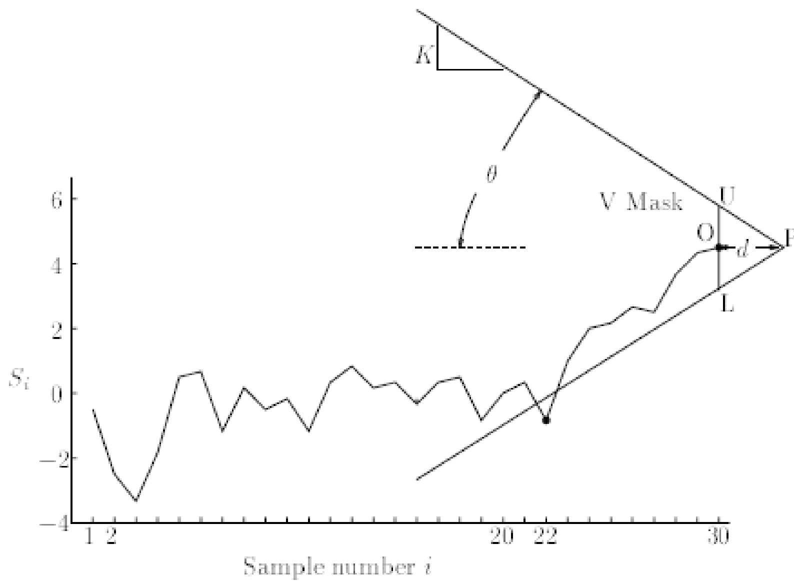
Example: Shewhart vs. CUSUM

Sample i	x_i	x_i-10	S_i
1	9.45	-0.55	-0.55
2	7.99	-2.01	-2.56
3	9.29	-0.71	-3.27
4	11.66	1.66	-1.61
5	12.16	2.16	0.55
6	10.18	0.18	0.73
7	8.04	-1.96	-1.23
8	11.46	1.46	0.23
9	9.20	-0.80	0.57
10	10.34	0.34	-0.23
11	9.03	-0.97	-1.20
12	11.47	1.47	0.27
13	10.51	0.51	0.78
14	9.40	-0.60	0.18
15	10.08	0.08	0.26
16	9.37	-0.63	-0.37
17	10.62	0.62	0.25
18	10.31	0.31	0.56
19	8.52	-1.48	-0.92
20	10.84	0.84	-0.08
21	10.40	0.40	0.32
22	8.83	-1.17	-0.85
23	11.79	1.79	0.94
24	11.00	1.00	1.94
25	10.10	0.10	2.04
26	10.58	0.58	2.62
27	9.88	-0.12	2.50
28	11.12	1.12	3.62
29	10.81	0.81	4.43
30	10.02	0.02	4.45



V Mask: Limits on Slope of CUSUM Chart

- Control V mask centered at each observation; if all previous S_i lie within the arms of the *V* mask, process is in control
- Sample 22 lies below the lower arm when mask centered at 30th sample ⇒ have detected upward shift in process mean
- Calculation of parameters d and θ of the *V* mask (see Montgomery)



V Mask Construction

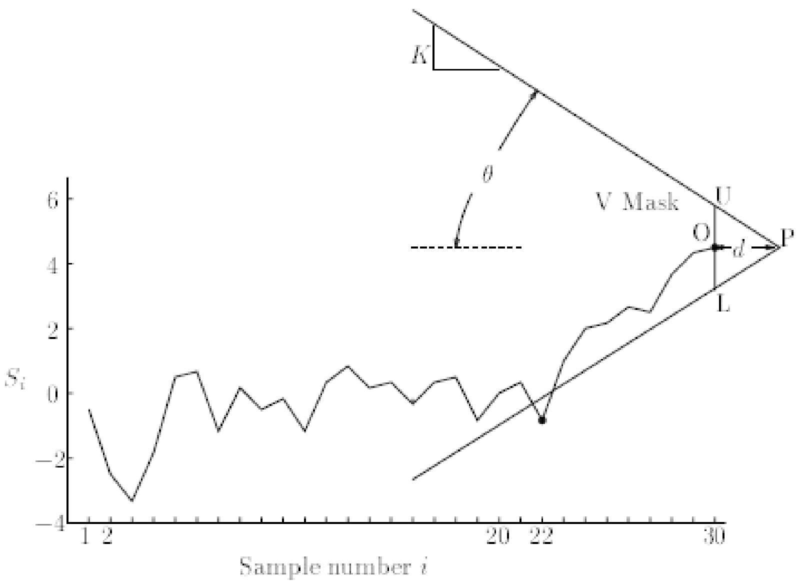
V mask is a function of

- Δ = magnitude of shift in $\bar{x}$ to be detected
- α = type 1 error

V mask construction

- Calculate

$$D = \frac{\Delta}{\sigma_{\bar{x}}} = \#\ \text{of standard deviations}$$



- $$d = -\frac{2}{D^2}\ln\left(\frac{\alpha}{2}\right)$$

- $$\theta = \tan^{-1}\left(\frac{\Delta}{2K}\right) \qquad K = \frac{\text{Vertical axis scale}}{\text{Horizontal axis scale}}$$

The EWMA Control Chart

- Plot z_j versus j: exponentially weighted moving average of samples upto the j^{th} sample

$$z_j = \lambda \bar{x}_j + (1-\lambda) z_{j-1}, 0 < \lambda \le 1$$

$$\text{where} \quad z_0 = \bar{\bar{x}}$$

- EWMA is weighted average of current and all past observations $\Rightarrow$ insensitive to normality assumption (central limit theorem)$\rightarrow$ideal control chart for individual observations (n = 1)
- If $\bar{x}_i$ are independent with variance σ^2 / n, variance of z_j is

$$\sigma_{z_j}^2 = \frac{\sigma^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2j}\right]$$

$$\lim_{j\to\infty} \sigma_{z_j}^2 = \frac{\sigma^2}{n}\left(\frac{\lambda}{2-\lambda}\right)$$

solve Lyapunov Equation:

$$\sigma_{z_j}^2 = (1-\lambda)^2 \sigma_{z_j-1}^2 + \lambda^2 \frac{\sigma^2}{n}$$

The EWMA Control Chart (cont'd)

- Control limits of EWMA chart (for large sample number *j*)

$$\text{UCL}=\bar{\bar{x}}+3\sigma\sqrt{\frac{\lambda}{(2-\lambda)n}}$$

$$\text{LCL}=\bar{\bar{x}}-3\sigma\sqrt{\frac{\lambda}{(2-\lambda)n}}$$

- If σ unknown, must be estimated from R-chart

$$\text{UCL}=\bar{\bar{x}}+3\frac{\bar{R}}{d_2}\sqrt{\frac{\lambda}{(2-\lambda)n}}$$

$$\text{LCL}=\bar{\bar{x}}-3\frac{\bar{R}}{d_2}\sqrt{\frac{\lambda}{(2-\lambda)n}}$$

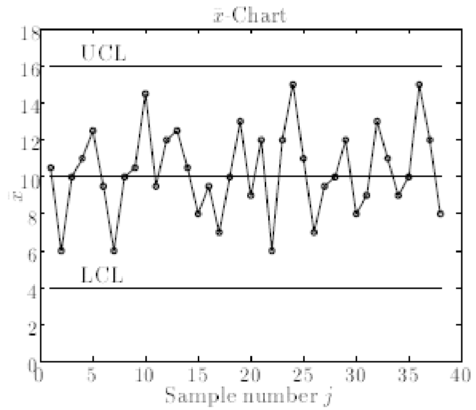
- Choice of λ and *k* (= 3 above) can be determined on the basis of ARL
  - Popular choices of λ : 0.08, 0.10, and 0.15 → use smaller λ to detect smaller shifts
  - Use *k* = 3 except when $\lambda \le 0.10$, use *k* = 2.75

EWMA Example

Construct EWMA chart from given $\bar{x}$-chart

Use λ = 0.2

Sample j	$\bar{x}_j$	z_j	LCL	UCL	Sample j	$\bar{x}_j$	z_j	LCL	UCL
1	**10.5**	**10.10**	**8.00**	**11.20**	**20**	**9.0**	**10.05**	**8.00**	**12.00**
2	**6.0**	**9.28**	**8.46**	**11.54**	**21**	**12.0**	**10.44**	.	.
3	**10.0**	**9.42**	**8.26**	**11.72**	**22**	**6.0**	**9.55**	.	.
4	**11.0**	**9.74**	**8.18**	**11.82**	**23**	**12.0**	**10.04**	.	.
5	**12.5**	**10.29**	**8.11**	**11.89**	**24**	**15.0**	**11.03**		
6	**9.5**	**10.13**	**8.07**	**11.93**	**25**	**11.0**	**11.00**		
7	**6.0**	**9.31**	**8.04**	**11.96**	**26**	**7.0**	**10.22**		
8	**10.0**	**9.45**	**8.03**	**11.97**	**27**	**9.5**	**10.08**		
9	**10.5**	**9.66**	**8.00**	**12.00**	**28**	**10.0**	**10.06**		
10	**14.5**	**10.62**	.	.	**29**	**12.0**	**10.45**		
11	**9.5**	**10.40**	.	.	**30**	**8.0**	**9.96**		
12	**12.0**	**10.72**	.	.	**31**	**9.0**	**9.77**		
13	**12.5**	**11.07**			**32**	**13.0**	**10.41**		
14	**10.5**	**10.96**			**33**	**11.0**	**10.53**		
15	**8.0**	**10.37**			**34**	**9.0**	**10.23**		
16	**9.5**	**10.19**			**35**	**10.0**	**10.18**		
17	**7.0**	**9.56**			**36**	**15.0**	**11.14**		
18	**10.0**	**9.64**			**37**	**12.0**	**11.32**		
19	**13.0**	**10.32**			**38**	**8.0**	**10.65**		



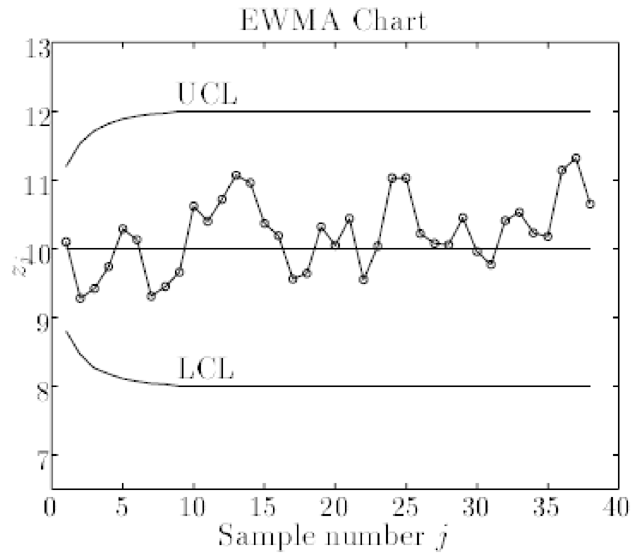
EWMA Example (cont'd)

- Control limits for EWMA chart

$$\text{UCL}=\bar{\bar{x}}+3\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}}=10.0+6.0\sqrt{\frac{0.2}{1.8}}=12.0$$

$$\text{Center line}=\bar{\bar{x}}=10.0$$

$$\text{LCL}=\bar{\bar{x}}-3\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}}=10.0-6.0\sqrt{\frac{0.2}{1.8}}=8.0$$



Capability Analysis Using a Control Chart: Example

Bursting strength of 20 samples of soft-drink bottles

Sample	Data					$\bar{x}$	R
1	265	205	263	307	220	252.0	102
2	268	260	234	299	215	255.2	84
3	197	286	274	243	231	246.2	89
4	267	281	265	214	318	269.0	104
5	346	317	242	258	276	287.8	104
6	300	208	187	264	271	246.0	113
7	280	242	260	321	228	266.2	93
8	250	299	258	267	293	273.4	49
9	265	254	281	294	223	263.4	71
10	260	308	235	283	277	272.6	73
11	200	235	246	328	296	261.0	128
12	276	264	269	235	290	266.8	55
13	221	176	248	263	231	227.8	87
14	334	280	265	272	283	286.8	69
15	265	262	271	245	301	268.8	56
16	280	274	253	287	258	270.4	34
17	261	248	260	274	337	276.0	89
18	250	278	254	274	275	266.2	28
19	278	250	265	270	298	272.2	48
20	257	210	280	269	251	253.4	70

$\bar{\bar{x}} = 264.06 \quad \bar{R} = 77.3$

Specification on bursting strength: LSL= 200

R-chart

$$\text{Center line} = \bar{R} = 77.3$$

$$\text{UCL} = \bar{R}\left(1 + \frac{3d_3}{d_2}\right) = 163.49$$

$$\text{LCL} = \bar{R}\left(1 - \frac{3d_3}{d_2}\right) = 0$$

$\bar{x}$-chart

$$\text{Center line} = \bar{\bar{x}} = 264.06$$

$$\text{UCL} = \bar{\bar{x}} + 3\frac{\bar{R}}{d_2\sqrt{n}} = 308.66$$

$$\text{LCL} = \bar{\bar{x}} - 3\frac{\bar{R}}{d_2\sqrt{n}} = 219.46$$

Capability Analysis Using a Control Chart (cont'd)

- Process parameters from the control charts

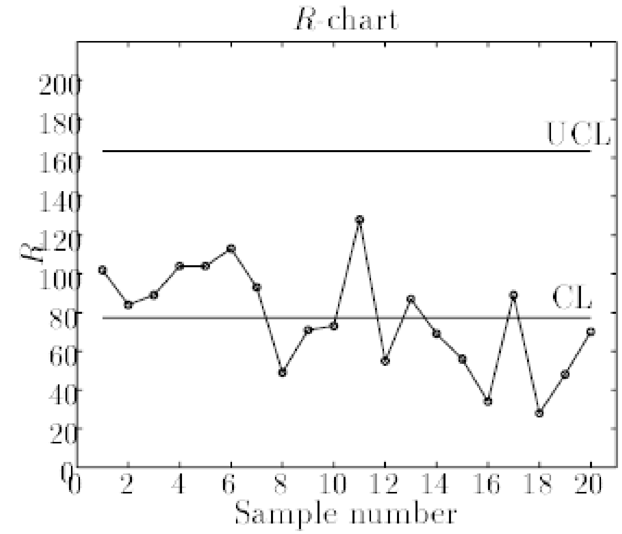
$$\hat{\mu} = \bar{\bar{x}} = 264.06$$

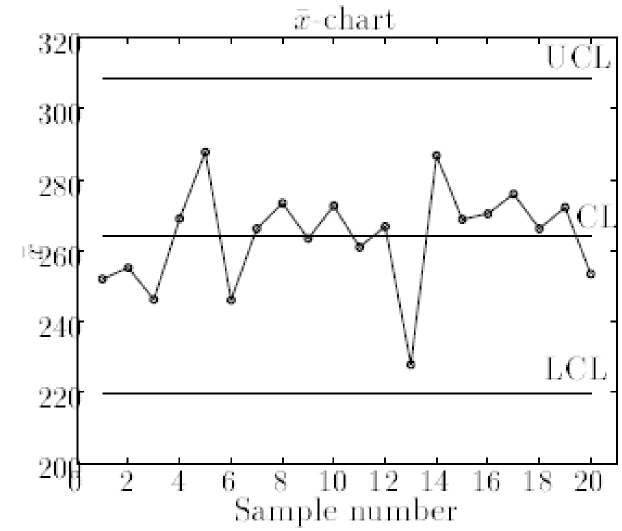
$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{77.3}{2.326} = 33.23$$

- One-sided process capability ratio

$$\mathrm{CP}_L = \frac{\hat{\mu} - \mathrm{LSL}}{3\hat{\sigma}} = \frac{264.06 - 200}{3(33.23)} = 0.64$$

- This CP inadequate (bottle strength is a safety factor) →process in control but operating at unacceptable level → management intervention required to improve the process





Evaluating Vendors using CP Analysis - 1

- Product Spec: 50mm ± 5mm
  - USL = 55mm; LSL = 45mm
- Vendor A: μ = 53mm; σ = 1.5mm

$$CP_k = \min(CP_L, CP_U) = \min(\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma})$$

$$= \min(\frac{2}{4.5}, \frac{8}{4.5}) = 0.44 \Rightarrow Bad$$

$$P\{45 \le y \le 55\} = 0.9082 \Rightarrow 9.18\% \; defects$$

- suppose shift mean μ to 50mm

$$CP_k = \min(CP_L, CP_U) = \min(\frac{5}{4.5}, \frac{5}{4.5}) = 1.11$$

$$P\{45 \le y \le 55\} = 0.9992 \Rightarrow 8 \; defects \text{ in } 10{,}000$$

Vendor B: μ = 52mm; σ = 0.6mm

$$CP_k = \min(CP_L, CP_U) = \min(\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma})$$

$$= \min(\frac{3}{1.8}, \frac{7}{1.8}) = 1.67 \Rightarrow Good$$

- suppose shift mean μ to 50mm

$$CP_k = \min(CP_L, CP_U)$$

$$= \min(\frac{5}{1.8}, \frac{5}{1.8}) = 2.67 \Rightarrow Excellent$$

$$P\{45 \le y \le 55\} = 0.9992 \Rightarrow 2 \; defects \text{ in } 1/\text{billion}$$

Even if mean = 51mm

$$CP_k = \min((\frac{4}{1.8}, \frac{6}{1.8}) = 2.2 \Rightarrow still \; Excellent$$

Evaluating Vendors using CP Analysis - 2

- Product Spec: 50mm ± 5mm
  - USL = 55mm; LSL = 45mm
- Vendor C: μ = 50mm; σ = 2.2mm

$$CP_k = \min(CP_L, CP_U)$$
$$= \min(\frac{5}{6.6}, \frac{5}{6.6}) = 0.76$$
$$P\{45 \le y \le 55\} = 0.9768 \Rightarrow 2.32\% \; defects$$

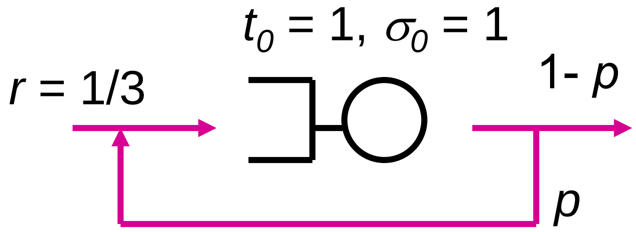
  - Need to reduce σ

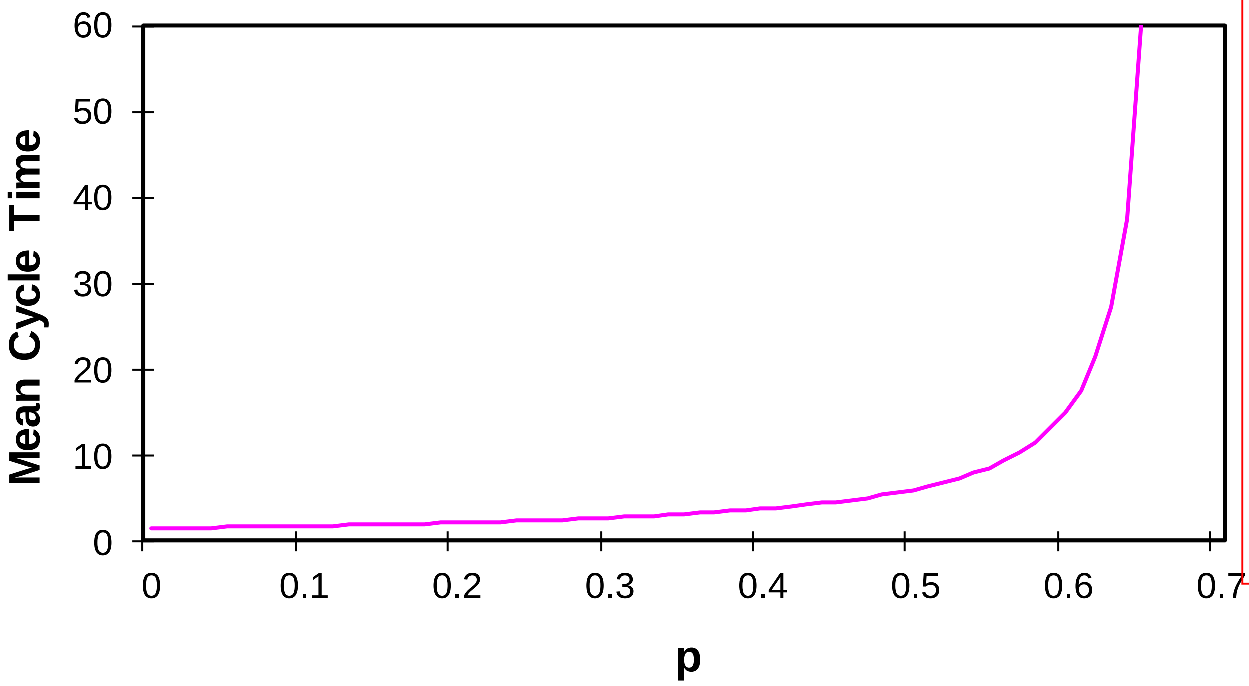
$$\sigma = 0.833 \Rightarrow CP_k = 2$$

Quality and Logistics

- Quality and Cost:
  - Cost increases with quality? (e.g., better materials)
  - Cost decreases with quality? (e.g., less correction cost)
  - Reality is a balance
- Quality Promotes Logistics:
  - ***Law: Variability degrades performance***
  - ***Law: Congestion effects increase nonlinearly with utilization***
  - Yield loss and rework are *major* sources of variability and lost capacity
- Logistics Promotes Quality:
  - Excess WIP obscures problems and delays/prevents diagnosis
  - Excess WIP magnifies losses
  - Excess cycle time degrades quality of service

Rework on a Single Station





$$t_e = \frac{t_0}{1-p}$$

$$\sigma_e^2 = \frac{\sigma_0^2}{1-p} + \frac{p t_0^2}{(1-p)^2}$$

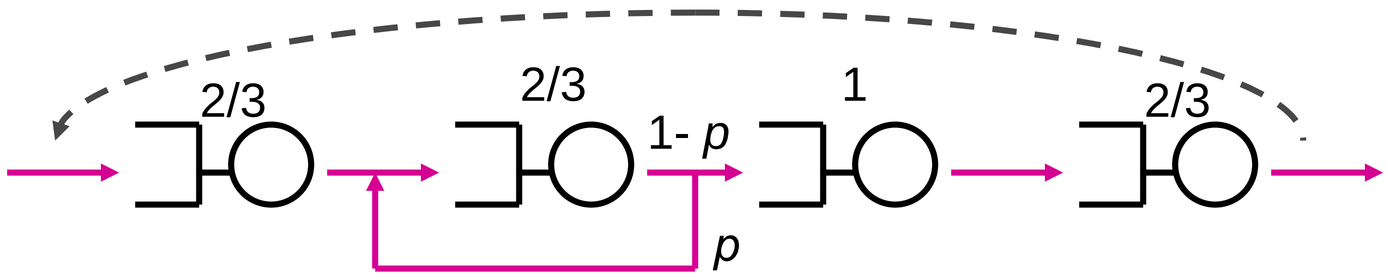
$$c_e^2 = c_0^2 + p(1-c_0^2)$$

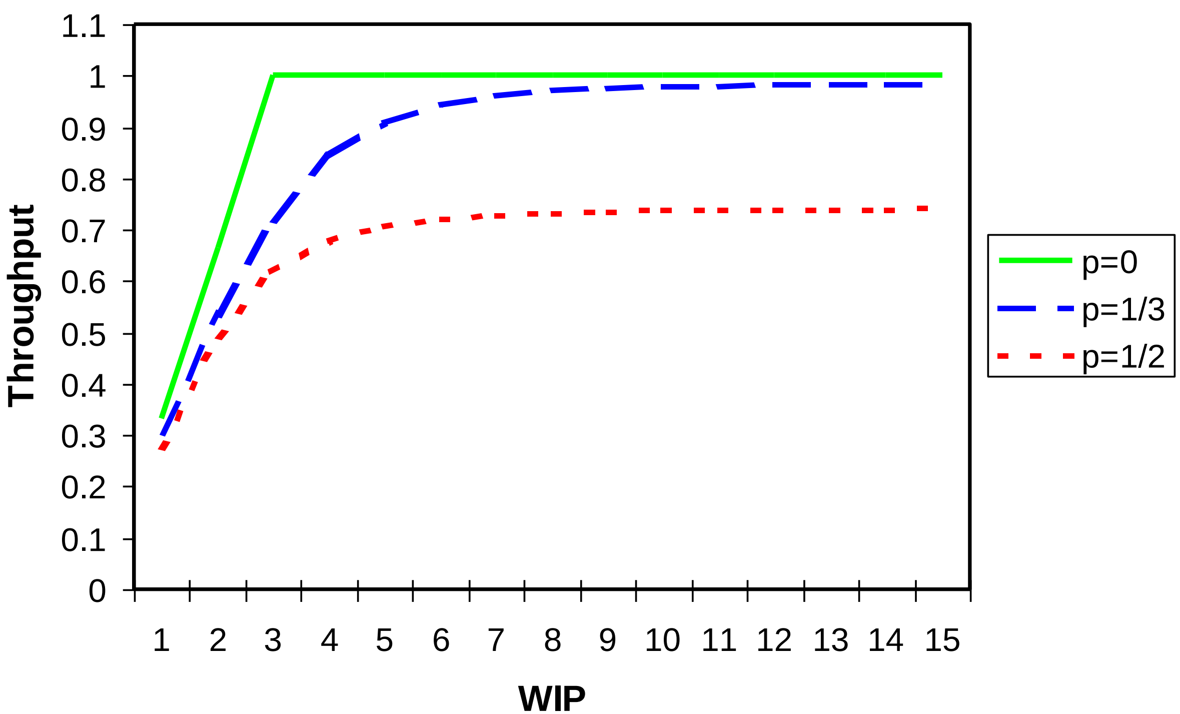
$$u = \frac{1}{3}\frac{t_0}{1-p}$$

$$CT = \frac{t_e}{1-u}$$

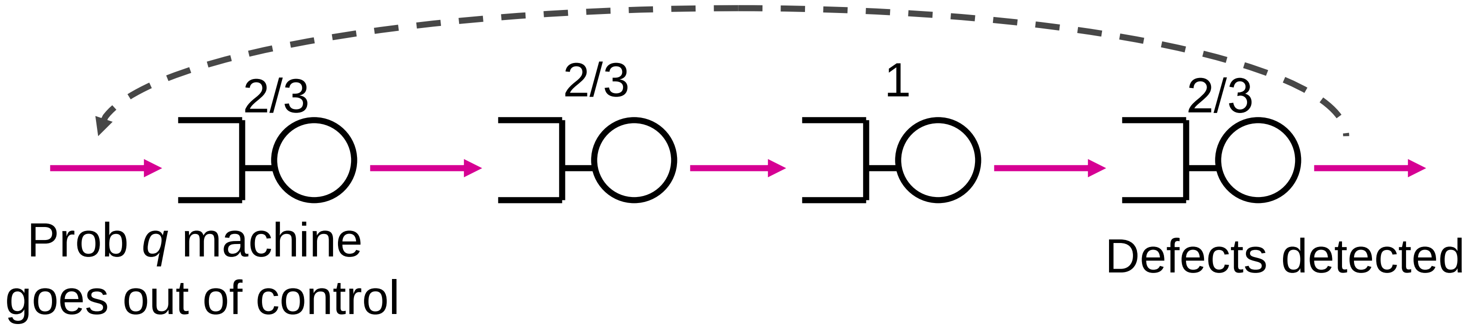
$$= \frac{t_0}{1-p-\frac{1}{3}t_0}$$

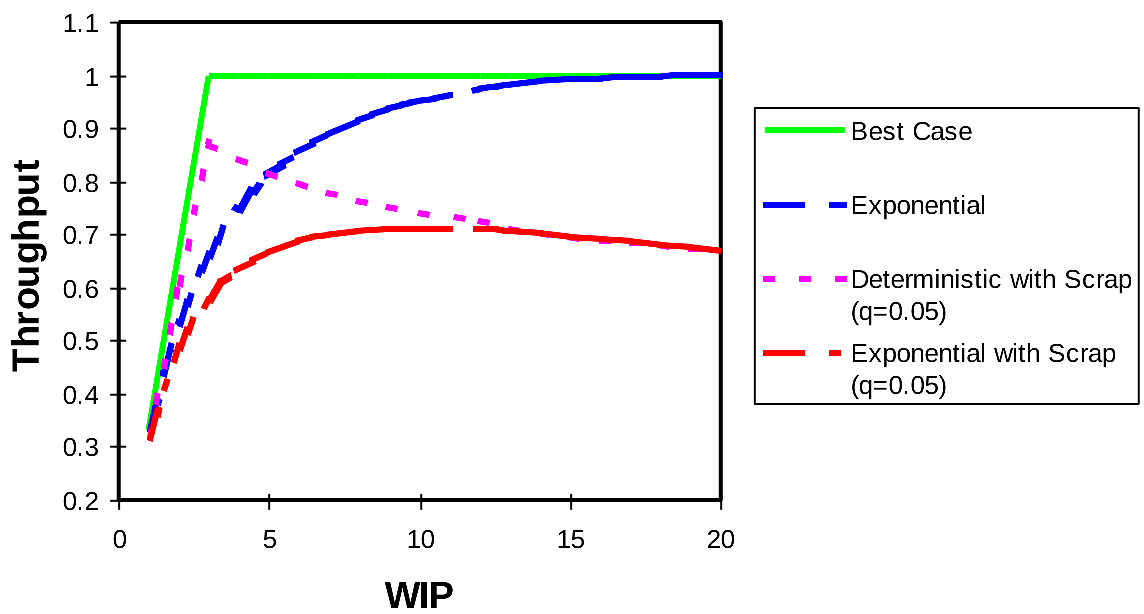
Rework in a Line





Defect Detection





Safety and Lead Times in Assembly Systems

- Required Service:
  - ***Single Component***: 95% service level
  - ***10 Component Assembly***: If each has 95% service level, then

$$\text{Prob}\{\text{All components arrive on time}\} = (0.95)^{10} = 0.5987$$

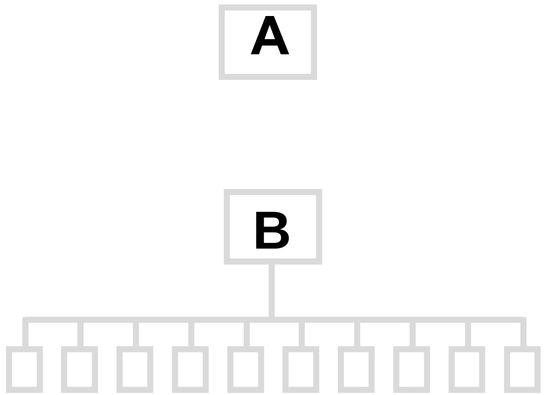
so to get 95% service on the assembly we need each component to have p% service, where

$$p^{10} = 0.95$$

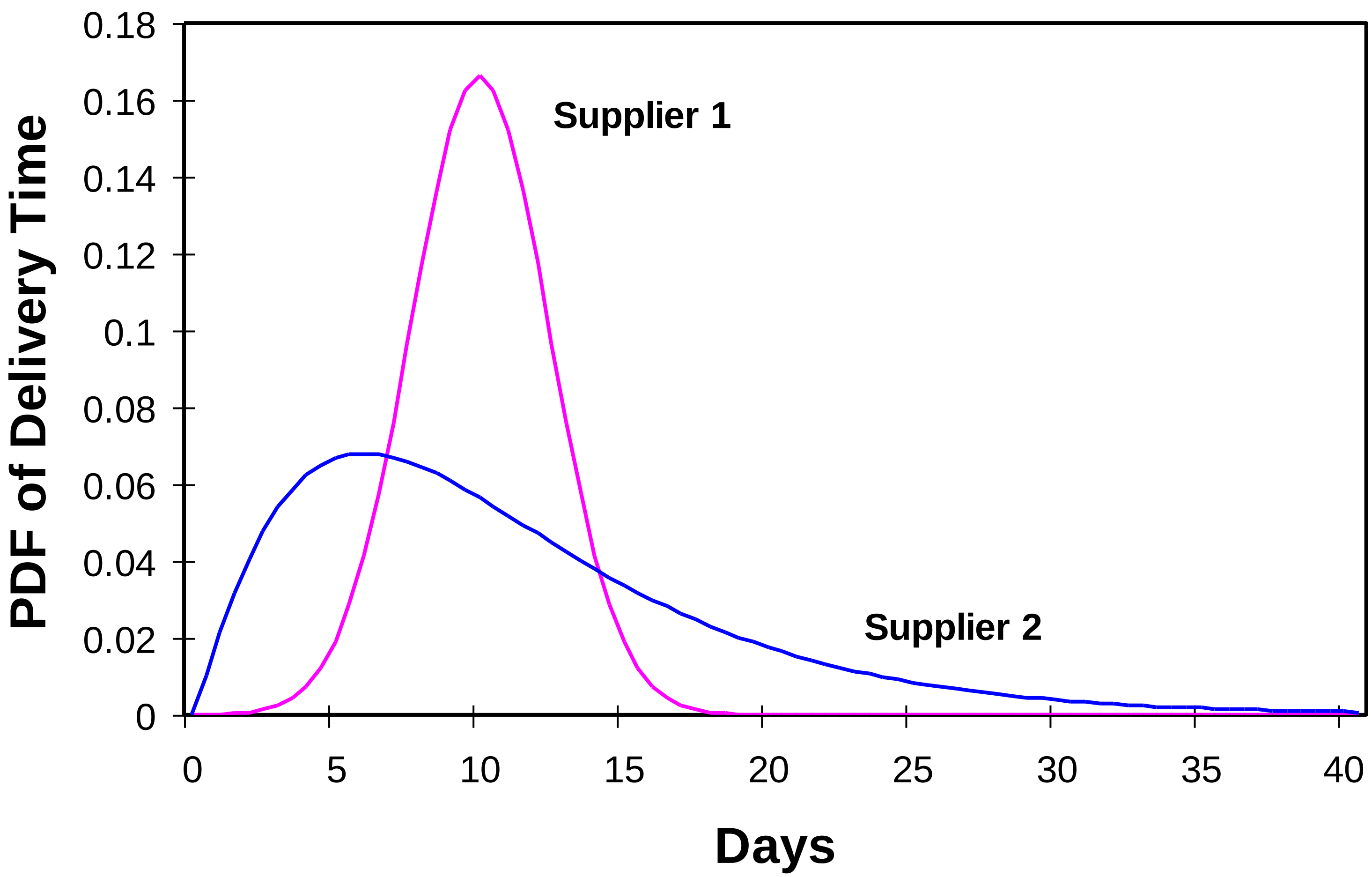
$$p = 0.95^{1/10} = 0.9949$$

Safety and Lead Times in Assembly Systems

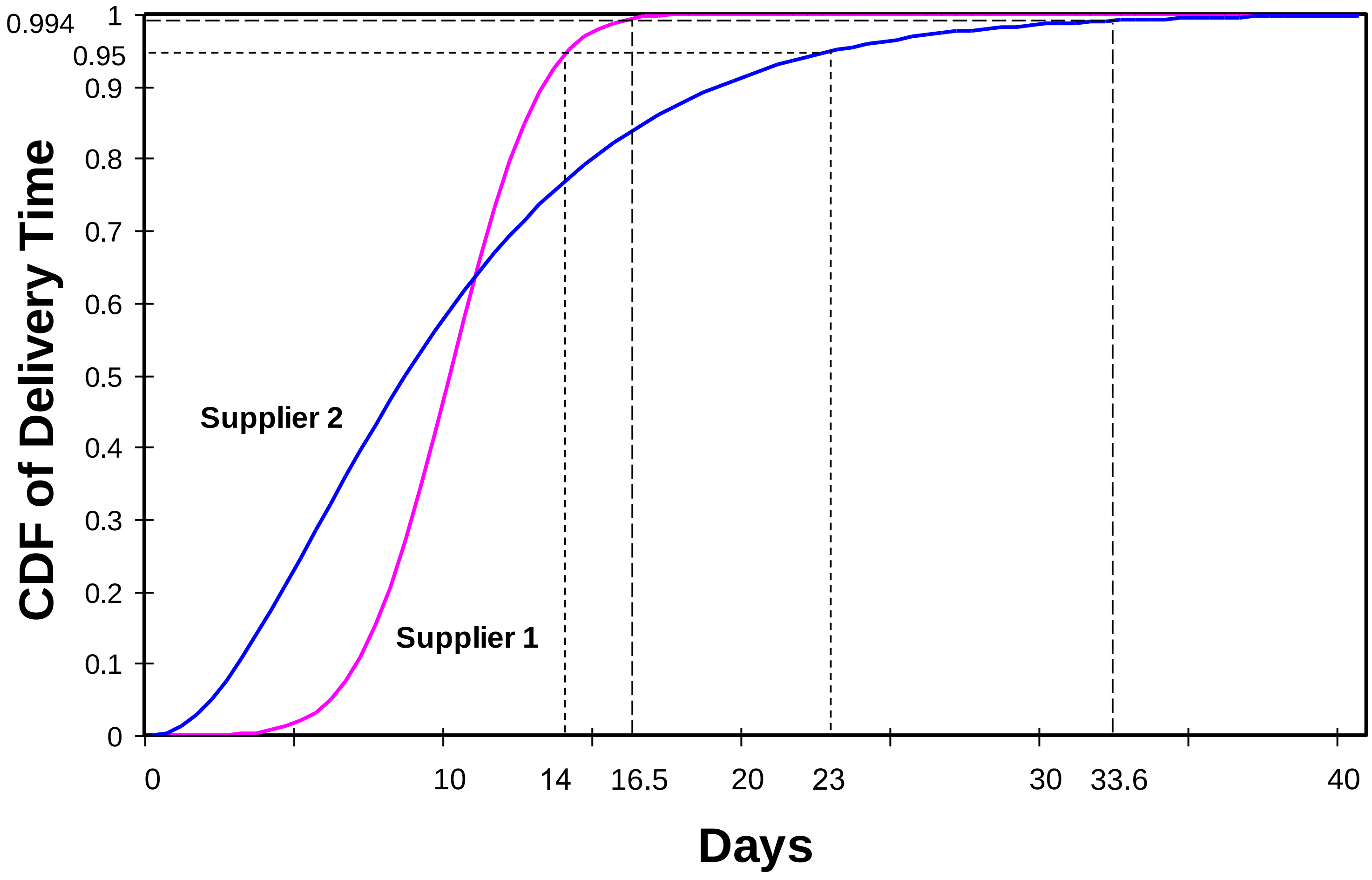
- Consequences:
  - *Single Component:*
    - Supplier 1: 14 day lead time
    - Supplier 2: 23 day lead time
  - *10 Component Assembly:*
    - Supplier 1: 16.3 day lead time
    - Supplier 2: 33.6 day lead time



Effect of Variability on Purchasing Lead Times



Effect of Variability on Purchasing Lead Times



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