Design for Quality

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ECE 6161

Modern Manufacturing System Engineering

Outline

- Attributes of Quality
- Measures of Quality Loss
- Robust Parameter Design (Taguchi)
- Response Surface Methods
- Multi-objective optimization

This Lecture

- On-line quality control (Next Lecture)
  - Statistical Process Control
  - On-line improvement

Attributes of Quality

- Quality Definitions:
  - ***Transcendent*: innate excellence or**
    "I know it when I see it" view.
  - ***Feature-based*: function of product attributes or**
    "more is better" view.
  - ***User-based*: customer satisfaction or**
    "beauty is in the eye of the beholder" view.
  - ***Manufacturing-based*: conformance to specifications,**
    related to "do it right the first time" view.
  - ***Value-based*: price/performance or**
    "affordable excellence" view.

Attributes of Quality (Cont'd)

- Customer Orientation:
  - Customer satisfaction depends on ***external*** quality
  - External quality depends on ***internal*** quality
  - Quality must address **product**, **process**, **system**
- Promoting Internal Quality:
  - Error prevention
  - Inspection improvement
  - Environment enhancement

Design for Quality

- Goal: obtain design parameter settings so that the least inherent variability is achieved
- Outline of topics
  1. Quality measures
  2. Design of experiments
  3. Taguchi's robust design
  4. Response surface methods
  5. Multiple quality characteristics (time permitting)
  6. Illustrations and case studies

1. Measures of Quality Loss

- Fraction defective = (# of rejects)/ (total # of parts)
  - leads to the use of a step loss function in the *tolerance interval*
- Quadratic loss function

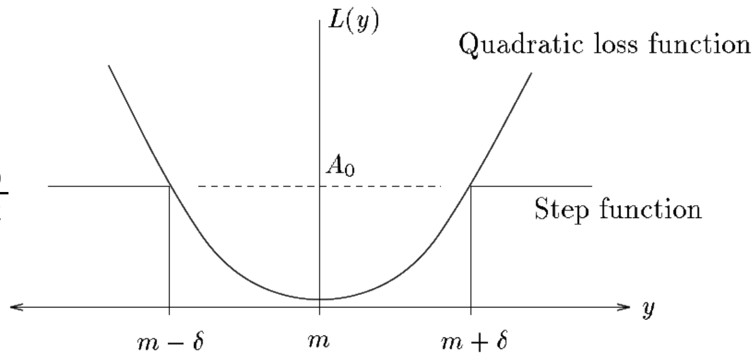
$$L(y) = k(y-m)^2$$

y : quality characteristic

m : target for y

  - results in smaller overall (expected) loss to society:

$$k = \frac{A_0}{\delta^2}$$

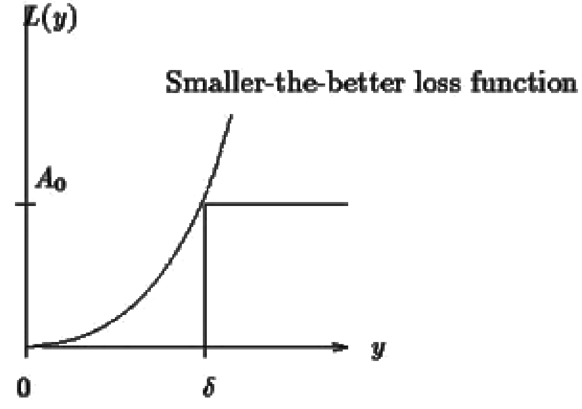


Other Loss Functions

- *Smaller-the-better* type quality characteristic

$$L(y) = ky^2$$

$$k = \frac{A_0}{\delta^2}$$

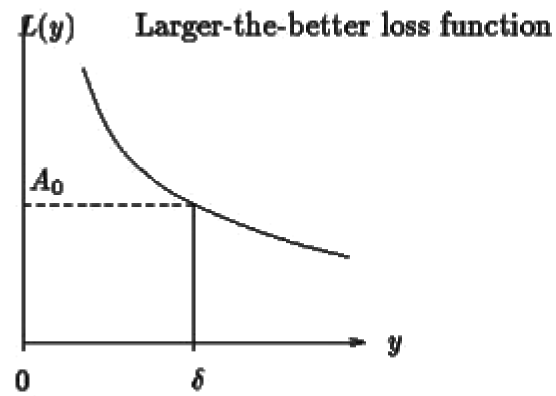


Example: Number of defects in a composite material part

- *Larger-the-better* type quality characteristic

$$L(y) = k\left[\frac{1}{y^2}\right]$$

$$k = A_0\delta^2$$



Example: strength of a part

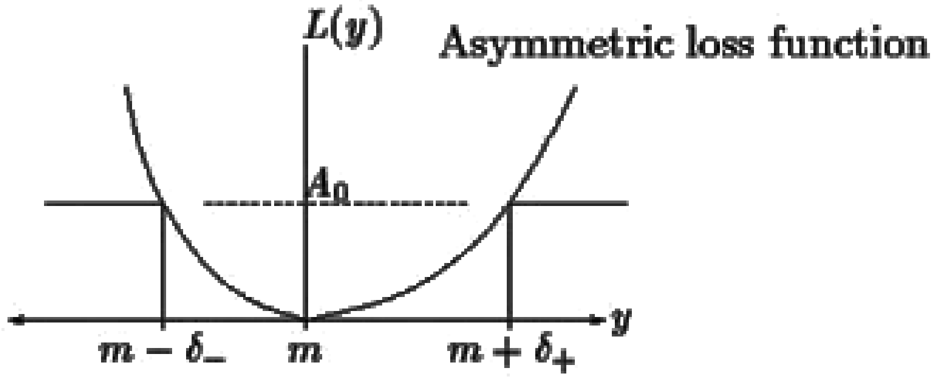
Other Loss Functions

- Asymmetric loss function

$$L(y)=\begin{cases}k_1(y-m)^2, & y>m\\ k_2(y-m)^2, & y\le m\end{cases}$$

$$k_1=\frac{A_0}{\delta_+^2}$$

$$k_2=\frac{A_0}{\delta_-^2}$$



2. Robust Design

- An *off-line* design technique: using experiments, find the settings of the product process parameters (*design parameters*) which minimize sensitivity of the quality characteristic to external uncontrollable variations (achieve *robustness*) — a.k.a. Taguchi's method

1. Identify a measure of variability (performance measure) that is a function of the design parameters, e.g., an average loss function, or a *signal-to-noise ratio*

2. Identify factors (variables) associated with the uncontrollable variation → noise factors – distinct from the design factors

3. Conduct experiment: systematically vary the design parameters as well as the noise factors to get estimates of the variability measure for chosen set of design parameter settings

4. Conduct data analysis to obtain process model and to identify the best design parameter settings (those that minimize the variability measure)

5. Run verification experiment to ensure that the 'best' design yields the expected improvement

Measure of Variability/Robustness

- Ideal measure for nominal-the-best type characteristic: the *expected quadratic loss function* (average quality loss per product)

$$
\begin{aligned}
\mathrm{MSE}(x) &= E\left[L\left(y(x)\right)\right] \\
&= kE\left[\left(y(x)-m\right)^2\right] \\
&= kE\left[\left(y(x)-\mu(x)+\mu(x)-m\right)^2\right] \\
&= k\underbrace{E\left[(y(x)-\mu(x))^2\right]}_{Variance}+2k\underbrace{E\left[(y(x)-\mu(x))\left(\mu(x)-m\right)\right]}_{=0}+k\underbrace{E\left[\left(\mu(x)-m\right)^2\right]}_{Bias^2} \\
&= k\sigma^2(x)+k\left(\mu(x)-m\right)^2
\end{aligned}
$$

Typical Bias-variance tradeoff in most modeling and learning problems

Objective: Minimize MSE(x) with respect to design parameters x

Surrogate Measure used by Taguchi

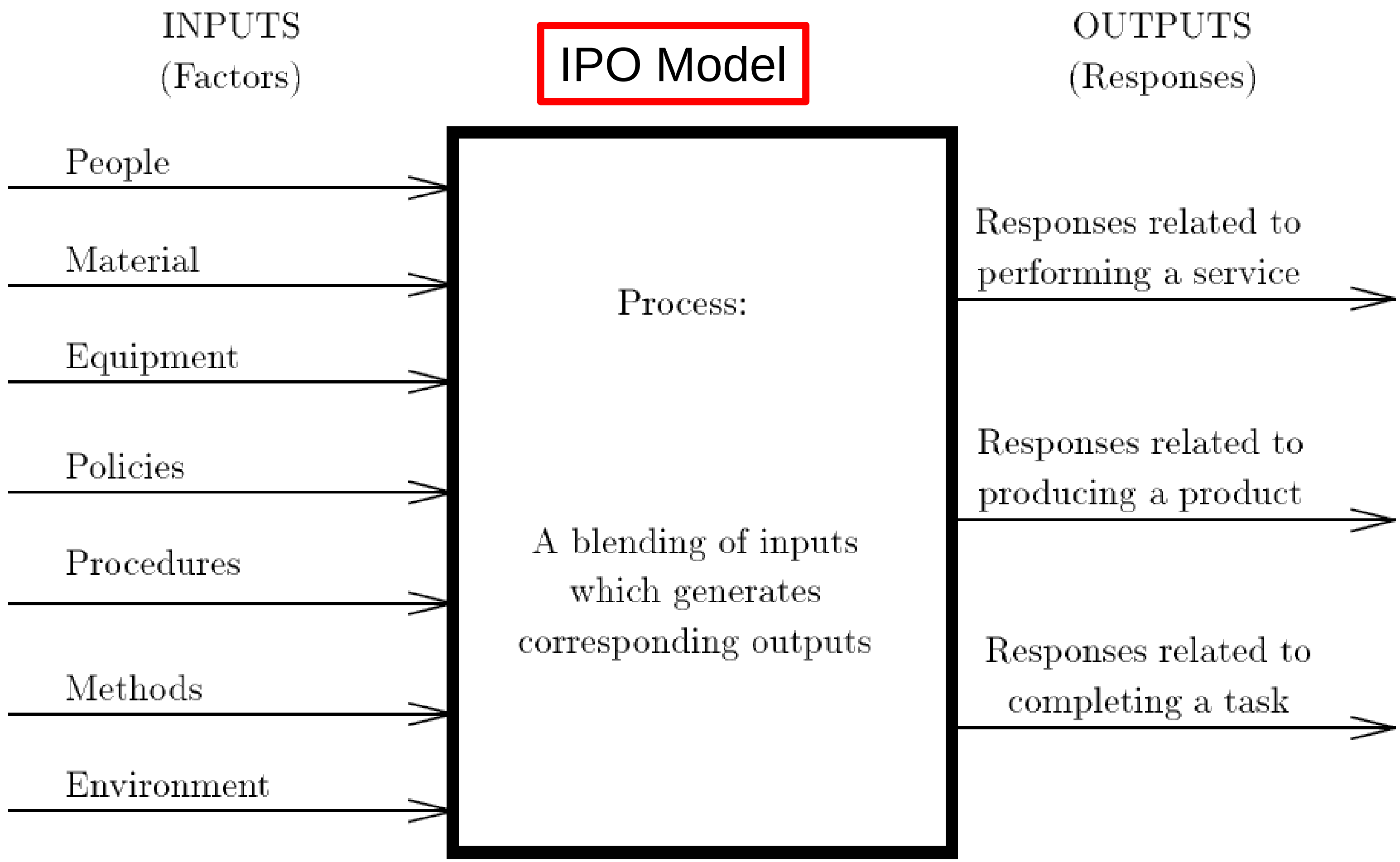
- Taguchi: instead of minimizing MSE(x), maximize a *signal-to-noise ratio*, e.g.,

$$SN_T(x) = 10\log_{10}\frac{\mu^2(x)}{\sigma^2(x)} = 20\log_{10}\frac{\mu(x)}{\sigma(x)}$$

1. Fix levels for each factor x_i, run an experiment at different combinations of factor-levels, x, measure $y(x)$ and $SN_T(x)$
2. Separate out signal factors from the other design factors (*control factors*)
   - *Signal factors*: affect the mean $\mu(x)$ of the response $y(x)$, but not $SN_T(x)$
3. Maximize $SN_T(x)$ with respect to control factors $\Rightarrow$ minimize $\sigma(x)$
4. Adjust signal factors to bring mean $\mu(x)$ close to target, m

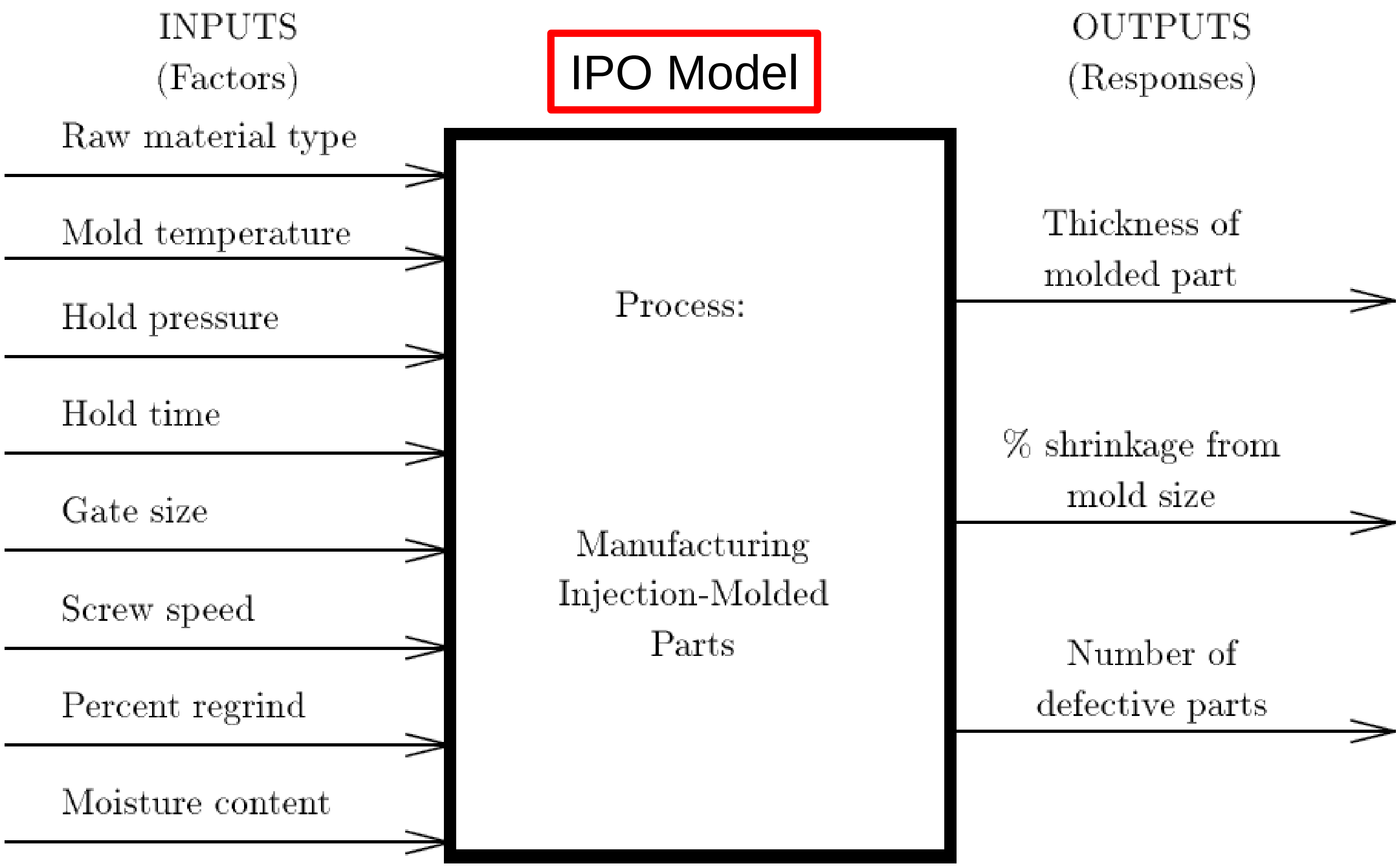
Role of Experimental Design

- Model a process: understand effects of inputs on outputs



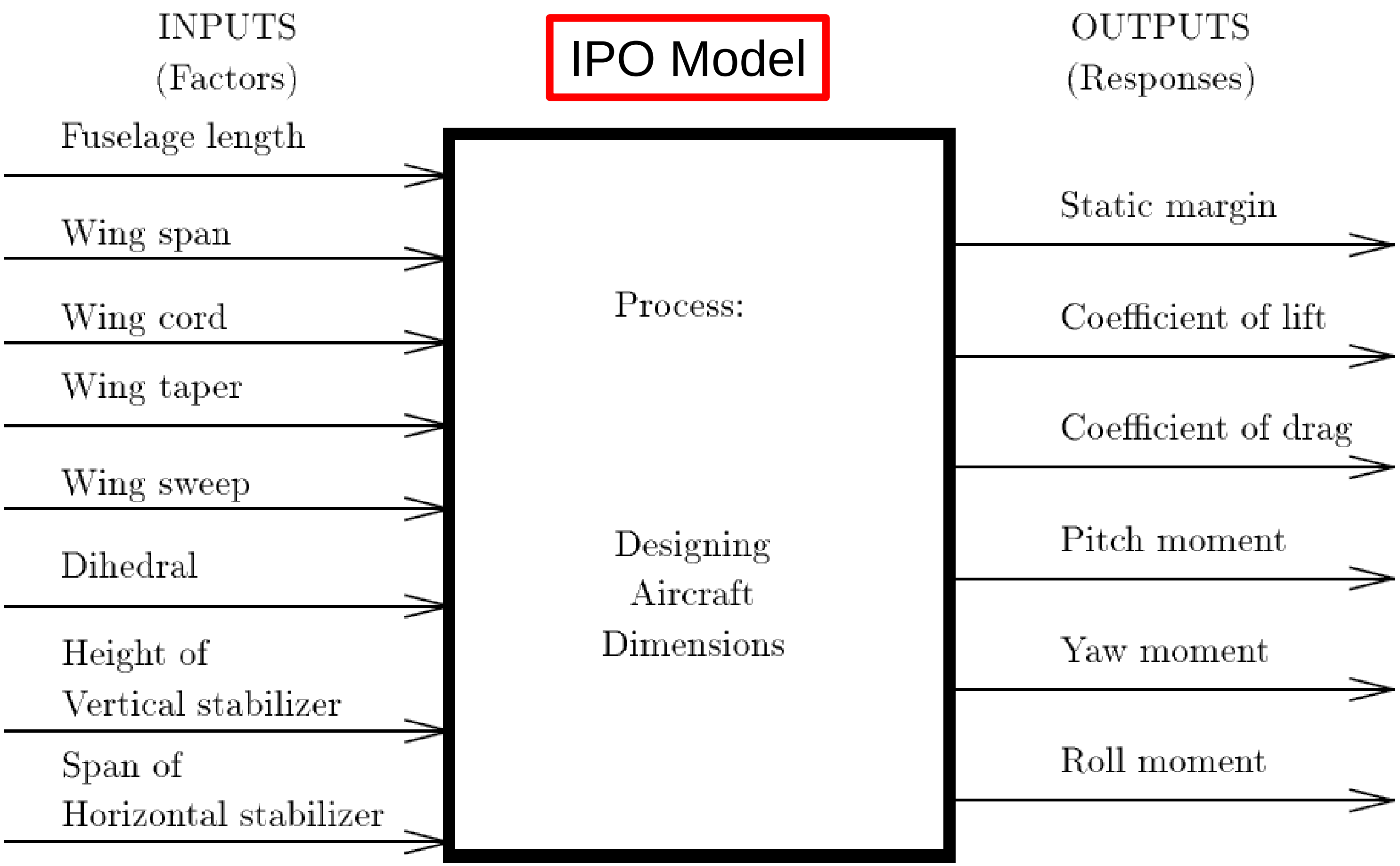
Examples of Processes - 1

- Injection molding process



Examples of Processes - 2

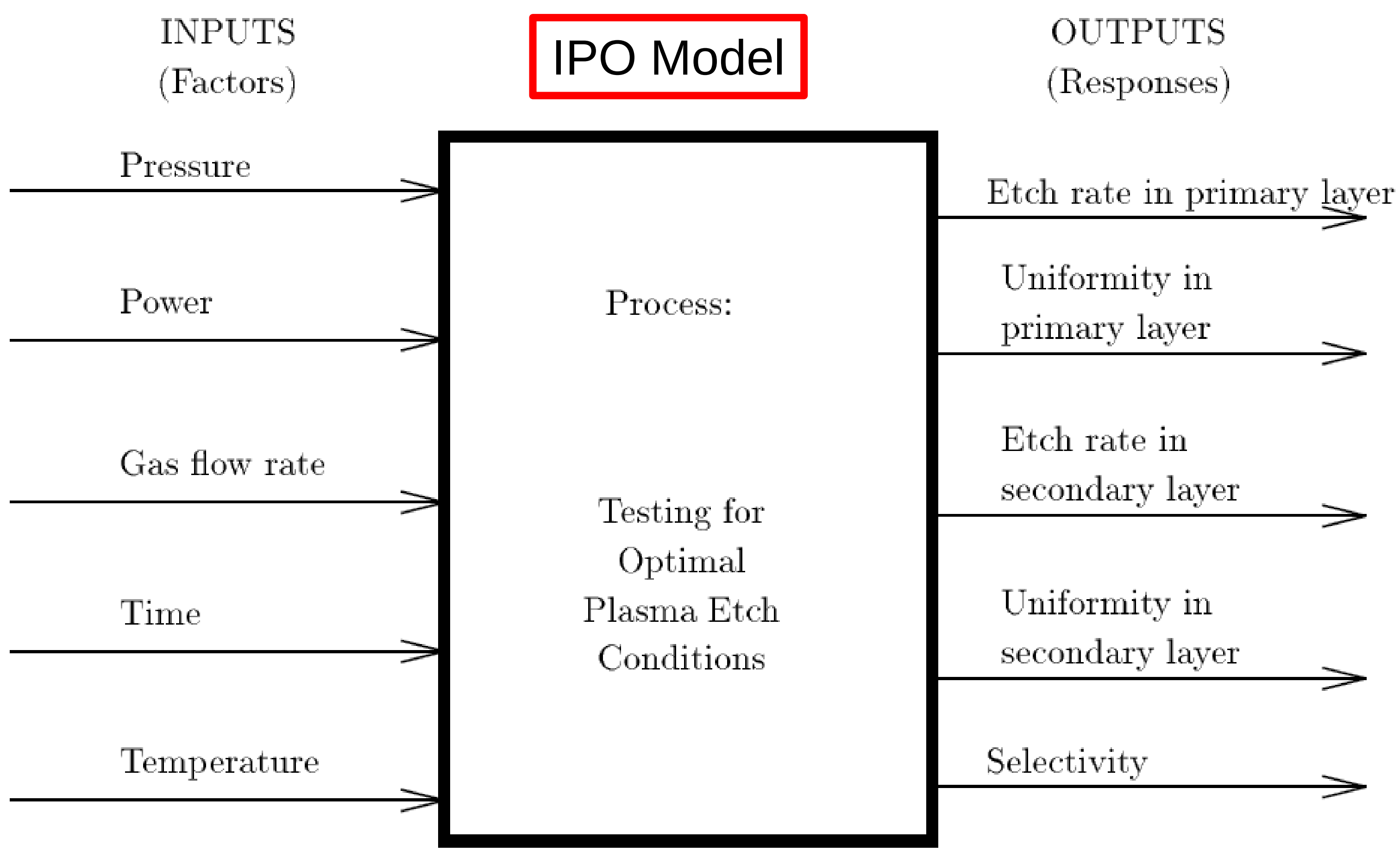
- Aircraft design



Examples of Processes - 3

- Semiconductor mfg: testing for optimal plasma etch conditions

IPO Model



Robust Design Example: Wave Solder Process

- Minimize the number of solder defects per million joints in a printed circuit board assembly plant

Controllable Factor	Levels	
	Low	High
S=Solder Pot Temperature	480°F	510°F
C=Conveyor Speed	7.2ft/m	10ft/m
F=Flux density	0.9°	1.0°
P=Preheat Temperature	150°F	200°F
W=Wave Height	0.5 in.	0.6 in.

Uncontrollable Factor	Levels	
Product Noise	Assembly 1	Assembly 2
Conveyor Speed Tolerance	-0.2ft/m	+0.2ft/m
Solder Pot Tolerance	-0.5°F	+0.5°F

Wave Solder Process: Inner and Outer Arrays

Inner Array

Run	Solder Pot Temperature	Conveyor Speed	Flux Density	Preheat Temperature	Wave Height
1	510	10	1	150	0.5
2	510	10	0.9	200	0.6
3	510	7.2	1	150	0.6
4	510	7.2	0.9	200	0.5
5	480	10	1	200	0.5
6	480	10	0.9	150	0.6
7	480	7.2	1	200	0.6
8	480	7.2	0.9	150	0.5

Outer Array

Noise Parameter	Replicate			
	1	2	3	4
Product Noise	Assembly#1	Assembly#1	Assembly#2	Assembly#2
ConveyorTolerance	-0.2	0.2	-0.2	0.2
SolderTolerance	-5	5	5	-5

Wave Solder Process: Experimental Data

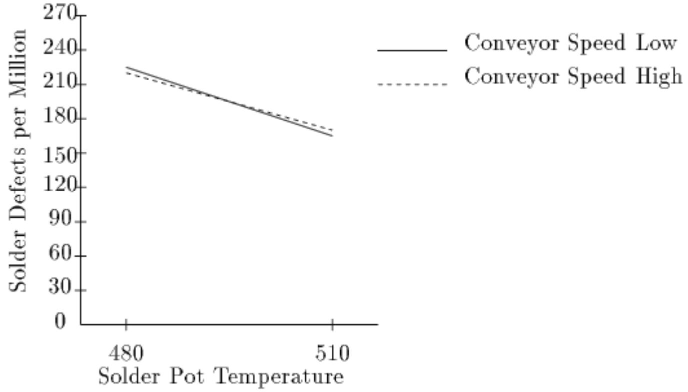
Outer array(L4)				
Parameter	1	2	3	4
ProductNoise	#1	#1	#2	#2
Conveyor Tolerance	-0.2	0.2	-0.2	0.2
Solder Tolerance	-5	+5	+5	-5

Inner Array (L_8)						Responses					
Run	Solder	Conveyor	Flux	Preheat	Wave	1	2	3	4	Mean	SN_S
1	510	10	1	150	0.5	194	197	193	275	215	-46.75
2	510	10	0.9	200	0.6	136	136	132	136	135	-42.61
3	510	7.2	1	150	0.6	185	261	264	264	244	-47.81
4	510	7.2	0.9	200	0.5	47	125	127	42	85	-39.51
5	480	10	1	200	0.5	295	216	204	293	252	-48.15
6	480	10	0.9	150	0.6	234	159	231	157	195	-45.97
7	480	7.2	1	200	0.6	328	326	247	322	305	-49.76
8	480	7.2	0.9	150	0.5	186	187	105	104	145	-43.59

- Inner array measures all five main effects (S,C,F,P,W) and interaction S- C
- SNR=-10$\log_{10}$(mean square defect count)

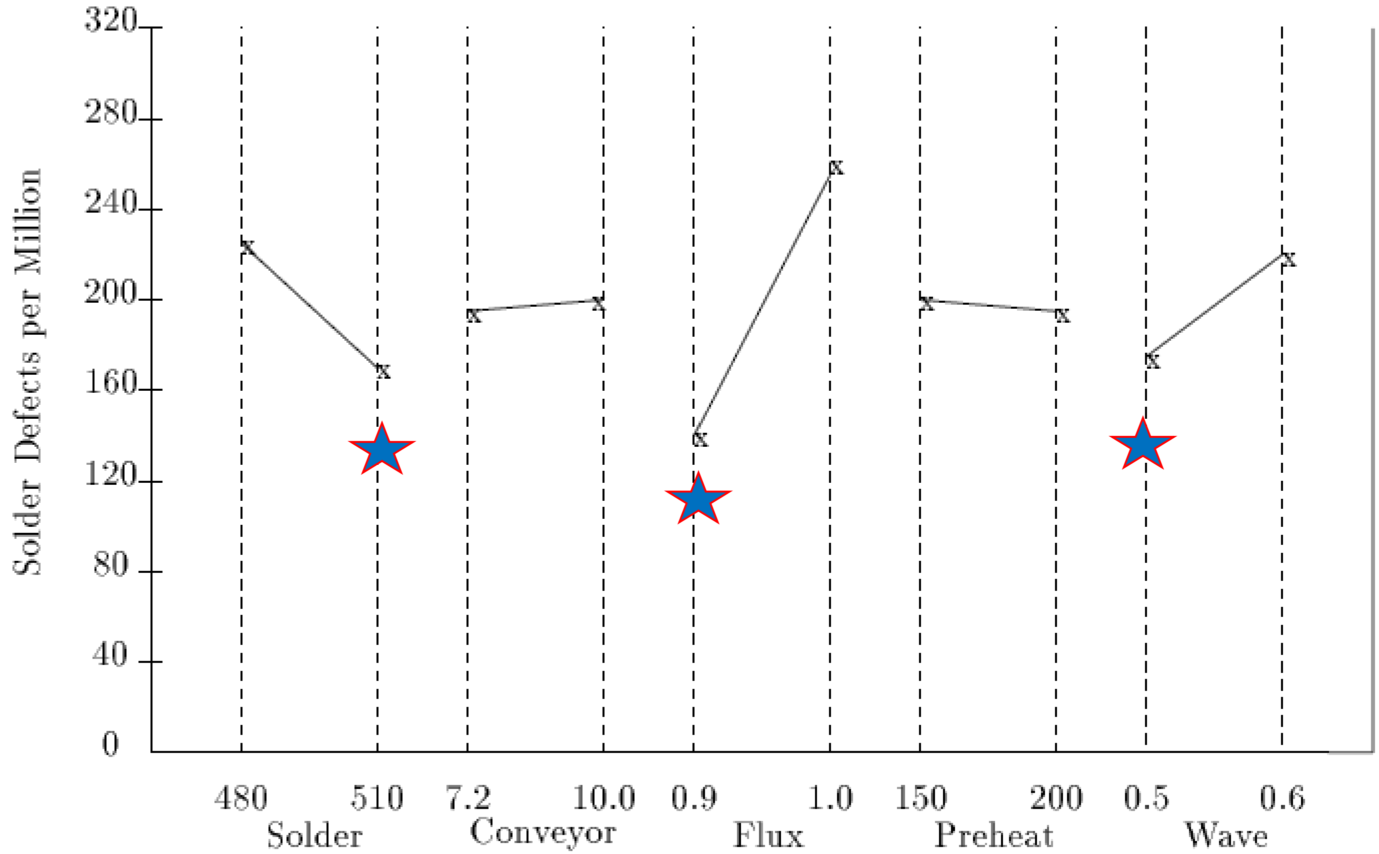
Wave Solder Process: Analysis of Means

Parameter	Level	Mean	SNR
Solder Pot temperature	480°F	225	-46.87
(S)	**510°F**	**170**	**-44.17**
Conveyor speed	**7.2**	**195**	**-45.17**
(C)	10	200	-45.87
Flux density	**0:9**	**140**	**-42.91**
(F)	1:0	255	-48.11
Preheat temperature	150°F	200	-46.03
(P)	**200°F**	**194**	**-45.01**
Wave height	**0.5in.**	**174**	**-44.5**
(W)	0.6in.	220	-46.54



- Interaction between solder pot temperature and conveyor speed insignificant

Wave Solder Process: Analysis of Means



- Only flux density and solder pot temperature significant for mean
- Wave height has moderate effect

Wave Solder Process: ANOVA Table for SNR

Parameter	d.f.	Sum of squares	Mean SS	F-Value
S	1	14.58	14.58	18.3
C	1	0.98	*	
F	1	54.08	54.08	67.89
P	1	2.08	*	
W	1	8.32	8.32	10.44
Residual	2	0.13		
Total	7	80.17		
Model (S+F+W)	3	76.98	25.66	32.22
Error (Residual+C+P)	4	3.19	0.7965	

$F_{3,4}(0.05) = 6.59;\ F_{1,4}(0.05) = 7.71;$

$R^2 = 76.98/80.1671 = 0.9602$

Prediction model:

$SNR = -45.52 + 1.35S - 2.60F - 1.02W$

Final Design:

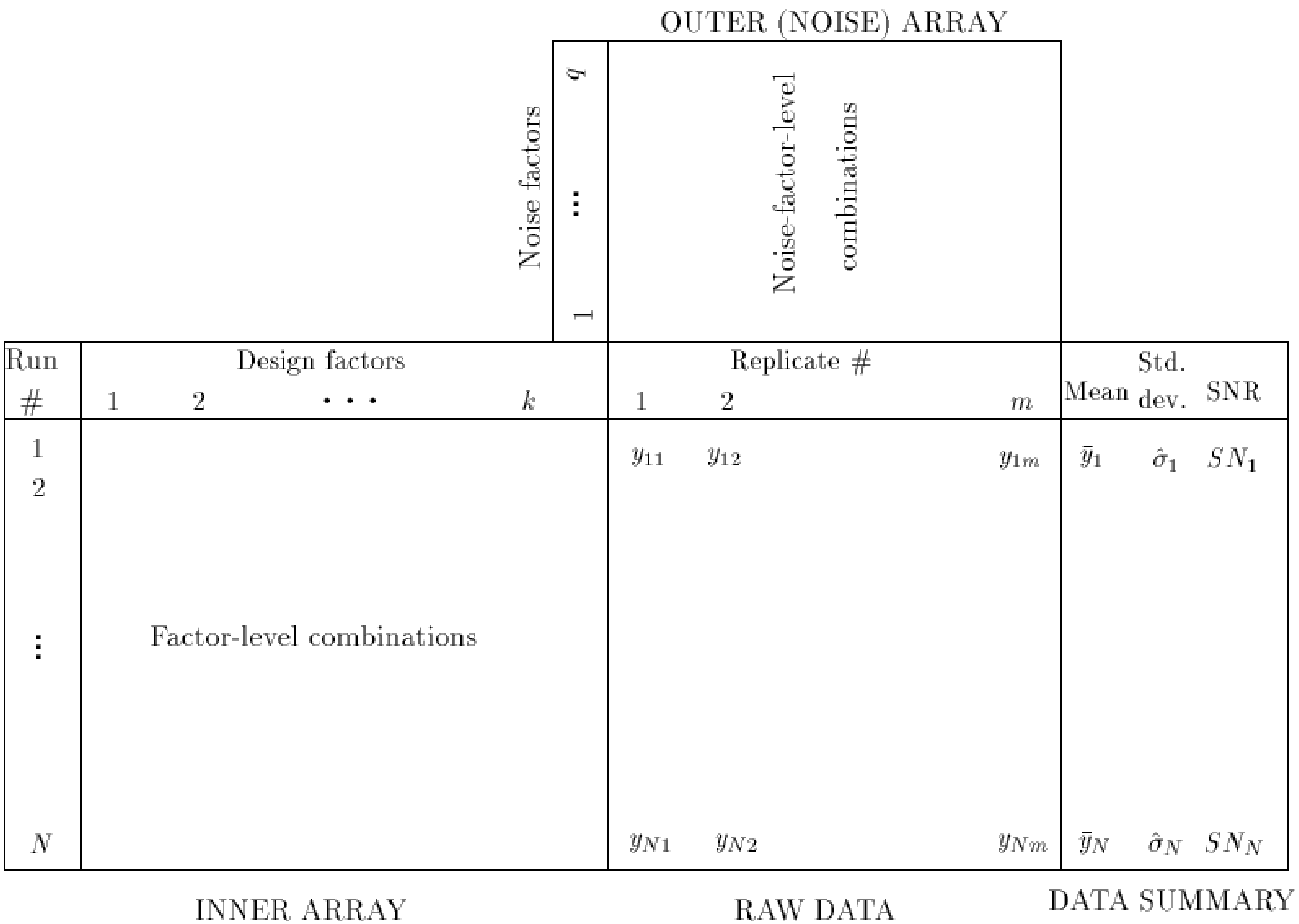
$S = 510;\ F = 0.9;\ W = 0.5;$

Predicted $SNR = -40.55$.

Components of Matrix Experiment

- Inner array: also known as *design array*
  - each row a unique combination of factor-levels
  - choice of design based on the theory of *design of experiments* or made from Taguchi's collection of *orthogonal arrays*
- Outer array: unique to Taguchi methods
  - each row a combination of noise-factor-levels
  - combinations span the range of possible variations of the noise factors that may occur in the (uncontrollable) field usage
  - each combination results in one measurement (replicate) of the response for every row of the inner-array
  - outer array design choice may again be one from Taguchi's collection of orthogonal arrays

Step 1: Design Matrix Experiment



Data Tabulation in Matrix Experiment

- Suppose N rows in design array $\Rightarrow$ N run combinations, m rows in outer array $\Rightarrow$ m replications
- Raw Data: m columns of measurements — one corresponding to each row of noise array
- Data Summary: averages of raw data for each row (run)
  - Let $y_{i1}, y_{i2}, :::, y_{im}$, be the raw measurements for i^{th} run
  - Average response: $\bar{y}_i = \frac{1}{m}\sum_{j=1}^{m} y_{ij}$
  - (Estimate of) Standard deviation of the response:

$$\hat{\sigma}_i = \sqrt{\frac{1}{m-1}\sum_{j=1}^{m}\left(y_{ij} - \bar{y}_i\right)^2}$$

  - (Estimate) of SNR (for nominal-the-best and larger-the-best type):

$$SN_T = 10\log_{10}\frac{\bar{y}_i^{\,2}}{\hat{\sigma}_i^{\,2}}$$

Inner Array Selection

- Objective: Choose inner array design so that
  - an adequate *functional relationship* between the variability measure (response) and the design factors is obtained
  - the number of runs is as small as possible
- Functional relationship (model): a sum of effects
  - *Main effect* of a factor: change in the response due to a change in level of that factor alone
  - *Interaction effect* of two or more factors: change in response due to the combined change in levels of those factors (after accounting for main effects)
- Model can be used to predict response at level-combinations not tested in the experiment

Inner Array Selection

- Factors affecting choice of inner array design
  - Number of levels each design factor is to take: decided by order of functional relationship
    - ❖ if factor effect linear: 2 levels
    - ❖ if factor effect quadratic: 3 levels
    - ❖ if factor is an attribute: # of levels = # of categories tested
  - Assumptions on whether the interaction effects are negligible or significant

Inner Array Design Types

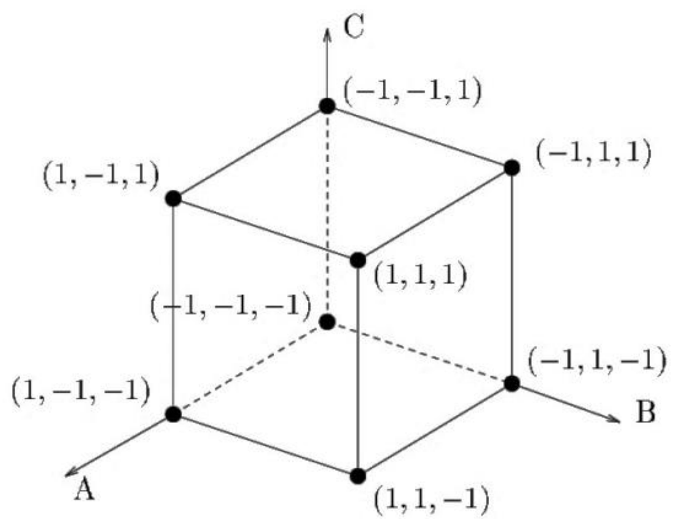
- **Full-factorial designs**: run all factor-level combinations
  - all main effects and their interactions can be measured
  - design size increases exponentially with number of factors and levels
- **Fractional-factorial designs**: obtained from full-factorial designs by assuming certain higher-order effect(s) are negligible → some interaction effects *aliased* with other effects
- **Taguchi's orthogonal arrays**: each column independently measures a main effect or an interaction; essentially a collection of
  - 2-level fractional-factorial designs
  - 2-level Plackett-Burman designs: for linear models with no interactions, or for screening experiments
  - 3-level fractional-factorial: for quadratic models with no (or few) interactions, or for qualitative factors with no (or few) interactions

Inner Array Selection (continued)

- Example: 3 factors
  - Factor A: 2 levels; Factor B: 2 levels; Factor C: 2 levels
  - Full-factorial design (all possible combinations) would require 2 x 2 x 2 = 8 runs → e.g., orthogonal array L_8

L_8

Expt. No.	Factor			Interaction effects			
	A	B	C	AB	BC	AC	ABC
1	-1	-1	-1	1	1	1	-1
2	-1	-1	1	1	-1	-1	1
3	-1	1	-1	-1	-1	1	1
4	-1	1	1	-1	1	-1	-1
5	1	-1	-1	-1	1	-1	1
6	1	-1	1	-1	-1	1	-1
7	1	1	-1	1	-1	-1	-1
8	1	1	1	1	1	1	1

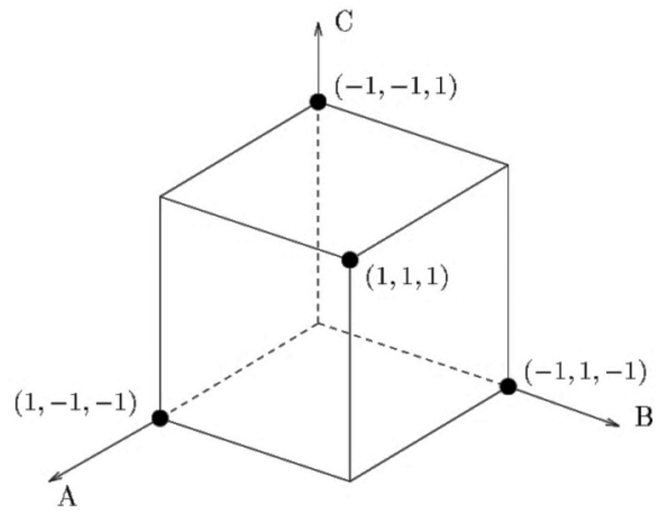


Inner Array Design (continued)

- Example (continued)
  - If no interactions among the three factor effects, can use a half-factorial design (only 4 runs) → orthogonal array L_4
  - Geometric interpretation: design points placed at only four appropriate vertices of cube instead of all eight

$L_4(2^{3-1})$

Expt. no.	Factor A	Factor B	Factor C
1	-1	-1	1
2	-1	1	-1
3	1	-1	-1
4	1	1	1



Defining relationship: I=ABC
Aliases: A=BC
B=AC
C=AB

Resolution of Fractional Factorial Designs

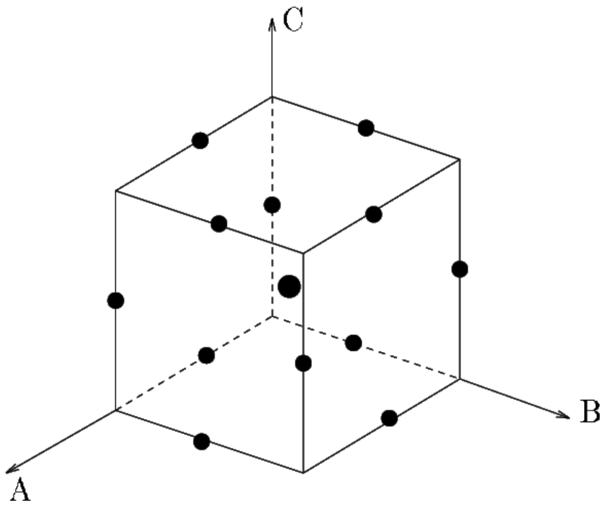
- **Aliasing of effects**: when two or more effects cannot be estimated separately of one another
  - In the L_4 array, the column for A actually estimates effect $A+BC \rightarrow A$ is aliased with interaction effect BC: $A = BC$
- Defining relationship: effect(s) aliased with overall mean (Why?)
  - Relationship $I = ABC \Rightarrow$ all runs correspond to ABC at level 1
- A design is of resolution *R* if no *p*-factor interaction effect (or main effect if p = 1) is aliased with another effect involving less than (R-p) factors, e.g.,
  - **Resolution III**: no main effect aliased with any other main effect, but a main effect can be aliased with a two-factor interaction
  - **Resolution IV**: no main effect aliased with any other main effect or with any two-factor interaction, but two-factor interactions may be aliased with each other

Box-Behnken Design

- Box-Behnken: 3-level designs ─ used for quantitative factors with three levels → useful for fitting models with
  - main (linear) effects: ±1 levels
  - quadratic effects: center points (0 level)
  - linear two-way interactions: two factors varied at a time; others held at zero

Box-Behnken for 3 factors

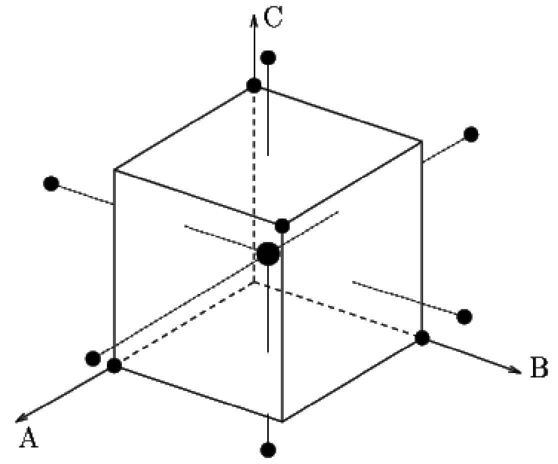
Factor A	Factor B	Factor C	No. of points
±1	±1	0	4
±1	0	±1	4
0	±1	±1	4
0	0	0	3



Box-Wilson Design

- Central-composite (Box-Wilson) designs: factors take 5 levels
  - for measuring main (linear) effects, quadratic effects, and linear interactions

Expt. No.	Factor A	Factor B	Factor C	
1	-1	-1	1	Factorial portion
2	-1	1	-1	
3	1	-1	-1	
4	1	1	1	
5	0	0	0	Center points
6	0	0	0	
7	0	0	0	
8	α	0	0	Axial points
9	-α	0	0	
10	0	α	0	
11	0	-α	0	
12	0	0	α	
13	0	0	-α	



  - Advantage over Box-Behnken designs: can have fewer runs if some quadratic effects and/or some interactions do not exist

Central-composite designs (continued)

- three portions: 2-level factorial portion; center-point portion; and axial-point portion
  - **2-level factorial portion**: full or fractional
  - **center-point portion**: number of center points of CCD
    - For 3 factors, $n_F = 8$ runs in factorial portion,
      - $n_C = 6 \longrightarrow$ uniform precision (prediction error variance at center is equal to that at unit distance from center)
      - $n_C = 3 \longrightarrow$ orthogonal design ($n_C = 4\sqrt{n_F + 1} - 2k,$ where k is the number of factors)
  - **axial-point portion**: $2k$ points. α chosen to ensure rotatability of design
    - Rotatability: prediction error variance depends only on distance from design center
    - For rotatability, $\alpha = (n_F)^{1/4}$

Other Design Types ...

- D-optimal designs: non-orthogonal, but very efficient
  - X: the design matrix
  - D-optimal design maximizes determinant of $X^T X$ (the information matrix)
  - Software packages available to assist in design generation
    - RS Discover (BBN Software Products)
    - SAS (SAS Institute Inc., Cary, NC)
    - E-Chip
    - Catalyst
- D-optimal designs are non-orthogonal; Box-Behnken and CCD designs are slightly non-orthogonal

  ⇒ analysis must be done via least-squares regression

Data Analysis for Orthogonal Arrays

- Analysis of Means: Computation of sample averages
  - Nm response measurements: N runs (rows), m replicates per run (columns)
  - Overall mean

$$\bar{y} = \frac{1}{Nm}\sum_{n=1}^{N}\sum_{k=1}^{m} y_{nk}$$

  - Average response in n^{th} trial

$$\bar{y}_n = \frac{1}{m}\sum_{k=1}^{m} y_{nk}$$

- **2-level arrays**: each column associated with one main effect or one interaction effect; p columns ⇒ p (main or interaction) effects
- **3-level arrays**: each column associated with two effects…

Analysis of Means (cont'd)

- Average response due to column *i* taking level *l*

$$\bar{y}_{i,l} = \frac{1}{q_i m} \sum_{\{n:l_i=l\}} \sum_{k=1}^{m} y_{nk},$$

$$i = 1, 2, \ldots. p; \;\; l = 1, 2, \ldots, L_i;$$

For 2-level experiments $L_i = 2$

$\{n : l_i = l\}$: set of indices denoting trial numbers in which column *i* takes level *l*.

q_i number of runs in which column *i* takes a given level *l*; $q_i L_i = N$.

- Effect measured by a 2-level column i = difference between the average responses due to the two levels of column *i*

$$\eta_i = \bar{y}_{i,2} - \bar{y}_{i,1}$$

Analysis of Means (cont'd)

- Predicted response for *n*-th run

$$\hat{y}_n = \bar{y} + \sum_{i=1}^{p} \left(\bar{y}_{i,l_{i(n)}} - \bar{y} \right)$$

$$l_{i(n)} = \text{level of column } i \text{ in } n\text{-th run}$$

- For 2-level experiments, also use *half-effects*

$$\text{Half effect}_i \ \frac{1}{2}\eta_i = \left(\bar{y}_{i,2} - \bar{y} \right) = -\left(\bar{y}_{i,1} - \bar{y} \right)$$

  - Predicted response for *n*-th run in this case can also be written as

$$\hat{y}_n = \bar{y} \pm (\text{sum of all half-effects})$$

$$'\pm' \Rightarrow - \text{ for level 1; } + \text{ for level 2}$$

ANOVA for Orthogonal Arrays

- Sums of squares (corrected for means)
  - Total sum of squares (SS) — measures overall variation in data

$$S = \sum_{n=1}^{N}\sum_{k=1}^{m}\left(y_{nk} - \bar{y}\right)^2$$

  - Sum of squares assignable to effect(s) measured by column i

$$S_i = mq_i \sum_{l=1}^{L_i}\left(\bar{y}_{i,l} - \bar{y}\right)^2, \quad i = 1, 2, \ldots, p$$

  - Sum of squares not explained by model (residual SS)
    - Sum of squares due to pure-error within-run variations or replication

$$S_{PE} = \sum_{n=1}^{N}\sum_{k=1}^{m}\left(y_{nk} - \bar{y}_n\right)^2,$$

    - Sum of squares due to lack of fit (if $p < N - 1$, i.e., all columns not used)

$$S_{LOF} = S - \sum_{i=1}^{p} S_i - S_{PE}$$

ANOVA (cont'd)

- Components of the sum of squares (SS)

$$
\begin{aligned}
S &= \sum_{n=1}^{N}\sum_{k=1}^{m}\left(y_{nk}-\bar{y}\right)^2 \\
&= \sum_{n=1}^{N}\sum_{k=1}^{m}\left(y_{nk}-\hat{y}_n+\hat{y}_n-\bar{y}\right)^2 \\
&= \sum_{n=1}^{N}\sum_{k=1}^{m}\left(y_{nk}-\hat{y}_n+\bar{y}+\sum_{i=1}^{p}\left(\bar{y}_{i,l_{i(n)}}-\bar{y}\right)-\bar{y}\right)^2 \\
&= \sum_{n=1}^{N}\sum_{k=1}^{m}\sum_{i=1}^{p}\left(\bar{y}_{i,l_{i(n)}}-\bar{y}\right)^2+\sum_{n=1}^{N}\sum_{k=1}^{m}\left(y_{nk}-\hat{y}_n\right)^2 \\
&= \sum_{i=1}^{p} mq_i\sum_{l=1}^{L_i}\left(\bar{y}_{i,l}-\bar{y}\right)^2+\sum_{n=1}^{N}\sum_{k=1}^{m}\left(y_{nk}-\hat{y}_n\right)^2 \\
&= \sum_{i=1}^{p} S_i + S_{RES} \\
&= (SS \text{ due to model}) + (\text{residual SS})
\end{aligned}
$$

ANOVA (cont'd)

- Residual SS = Sum of squares due to pure error + Sum of squares due to lack of fit

$$S = \sum_{n=1}^{N}\sum_{k=1}^{m}\left(y_{nk} - \hat{y}_n\right)^2$$

$$= \sum_{n=1}^{N}\sum_{k=1}^{m}\left(y_{nk} - \bar{y}_n + \bar{y}_n - \hat{y}\right)^2$$

$$= \sum_{n=1}^{N}\sum_{k=1}^{m}\left(y_{nk} - \bar{y}_n\right)^2 + \sum_{n=1}^{N}\sum_{k=1}^{m}\left(\hat{y}_n - \bar{y}_n\right)^2$$

$$= SS_{PE} + S_{LOF}$$

- Degrees of freedom (d.f)
  - Total d.f.= Nm
  - d.f. associated with the model: p (1 d.f. for each effect)
  - d.f. associated with residual = $Nm - p - 1$
    - d.f due to lack of fit = $Nm - p - 1$ (if $p < N - 1$ else 0)
    - D.f due to pure error = $Nm - N$

ANOVA Table

Source	d.f.	Sum of squares (SS)	Mean SS = SS/df	F-value
Effect i	$\cdots$ $d_i = L_i - 1$ $\cdots$	S_i	$MS_i = \frac{S_i}{d_i}$	$F_i = \frac{MS_i}{MSE}$
Model	$p = \sum_i d_i$	$S_M = \sum_{i=1}^{p} S_i$	$MS_M = \frac{MS_M}{p}$	$F_M = \frac{MS_M}{MSE}$
Residual	$Nm - p - 1$	S_{RES}	$MSE = \frac{S_{RES}}{Nm-p-1}$	
Total	$Nm - 1$	S		

- R^2 - statistic — fraction of variation about the overall mean explained by the fitted model $R^2 = \frac{S_M}{S}$

Inference from ANOVA

- Significance of model

$$F_M \sim F_{p,Nm-p-1} \ : \ F-\text{distribution with } p \text{ and } Nm-p-1 \text{ d.f.}$$

  - Model significant if $F_M > F_{p,Nm-p-1}(\alpha)$

    $F_{p,Nm-p-1}(\alpha)$: value of F-distribution with degrees of freedom p and $F_{p,Nm-p-1}$ for tail probability α

- Effect *i* significant at level α if $F_i > F_{d_i, Nm-p-1}(\alpha)$
- Sums of squares of insignificant factors are pooled with S_{RES} (error residual sum of squares) and the *F* -values are recomputed
- Only significant effects included in the prediction model
- Model with *p* significant effects:

$$\hat{y}_{l_1,l_2,\ldots,l_p} = \bar{y} + \sum_{i=1}^{p}\left(\bar{y}_{i,l_i} - \bar{y}\right)$$

Comments/Critique

- Assumptions in Robust Design
  - **Factor effects are additive**: interactions can either be eliminated or ignored by appropriate choice of factors and design matrix ⇒ best factor levels can be selected independently of each other
  - **Separability of signal factors and control factors is achievable**
  - **Use of outer array can give reliable estimates of dispersion**
- Problems
  - SNR measures do not always relate to the quadratic loss function
  - Separability of design factors not always achievable
  - Effort to ignore interactions results in crude models
  - Pooling of effects in error may make other unpooled, but insignificant, effects significant → biased, misleading model
  - Use of outer arrays to measure dispersion suspect

Improvements/Alternatives

- Use data transformations to achieve additivity and/or separability
- Use response surface methods to prevent ignoring of interactions
- Use dual objective approach (of achieving mean on target and minimizing variability) instead of maximizing SNR
- ANOVA: Use normal probability plots instead of pooling to identify significant effects
- Use outer array designs other than orthogonal arrays

Summary of Taguchi Methods

- An effective tool for quality improvement
  - simple philosophy ─ minimize variability
  - obvious approach ─ use experiments
- Reports of success even with crude and sometimes faulty statistical methods
  - highlights the absence of quality improvement effort in current industry
  - justifies ongoing research to further improve upon the methods
- More widespread acceptability possible if sounder experimental design principles are incorporated, e.g., use classical approach of sequential experimentation combined with the objective of variability reduction

3. Response Surface Methods (RSM)

- For quantitative factors; qualitative factors taken into account via blocking
- Used more often for process optimization than for product design
- Additivity and separability of factor effects not required unlike Taguchi's methods
- Quadratic loss function *M*(x) can be directly minimized by RSM instead of using Taguchi's two-step approach

Steps in RSM

- Design and conduct of experiment
- Data analysis
  - ANOVA: identify significant main- and interaction-effects
  - *Regression modeling:* estimate model parameters → response surface
  - Search response surface for optimum design point
- Approach and assumptions
  - Variance of response does not vary with design point (homoscedasticity)
  - Noise variables treated as nuisance factors: taken into account via *blocking*: separate response surfaces obtained for each block
- Mainly for modeling and optimizing mean of the response

Regression Modeling

- Highlights
  - Model the mean of the response as a polynomial in design factors
  - Select significant terms in polynomial model via ANOVA
  - Estimate model parameters (polynomial coefficients) using the least-squares method
- Notation
  - mean response denoted by η
  - k factors: levels denoted by the vector $\boldsymbol{x} = [x_1, \ldots, x_k]'$
  - Functional relationship $\eta(\boldsymbol{x})$ determined from measurements

$$y(x) = \eta(x) + \varepsilon$$

Regression Modeling: Notation

- Notation (continued)
  - mean response denoted by η
  - k factors: levels denoted by the vector $\boldsymbol{x} = [x_1, \ldots, x_k]^T$
  - Functional relationship $\eta(\boldsymbol{x})$ determined from measurements

$$y(x) = \eta(x) + \varepsilon$$

  - True function form

$$\eta(x) = \theta^T z(x)$$

$$z(x) = [1\, x_1 \cdots x_k\; x_1^2, \cdots x_k^2\; x_1 x_2 \cdots]^T$$

$\theta : p \times 1$ vector of polynomial coefficients to be estimated

- measurements at N distinct points $x_1, \ldots, x_N$

$$y = X\theta + e$$

  - X: $N \times p$ design matrix; i^{th} row of X: $z^T(x_i)$

Regression Modeling: Least-squares Estimation

- Estimate θ by $\hat{\theta}$ that minimizes the total squared prediction error

$$\left(y-\hat{y}\right)^T\left(y-\hat{y}\right)$$

$$\text{where } \hat{y} = X\hat{\theta}$$

- Least-squares estimate given by

$$\hat{\theta} = \left(X^T X\right)^{-1} X^T y$$

- Prediction model: mean response at any x

$$\hat{\eta}\left(x\right) = \hat{\theta}^T z\left(x\right)$$

Example: Plastic-Injection-Molding Process Optimization

- Obtain process variable settings to minimize variability in part shrinkage
- Process variables (design factors):
  - Mold temperature (x_1)
  - Screw speed (x_2)
  - Holding time (x_3)
- Response: Log of standard deviation of parts shrinkage (y)
  - Fit second-order model:

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_{11} x_1^2 + b_{22} x_2^2 + b_{33} x_3^2 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{23} x_2 x_3$$

- Use central composite design

Example (cont'd): Central Composite Design

	Mold Temperature X_1	Screw Speed x_2	Holding Time]x_3	Log Standard Deviation of Parts Shrinkage
Factorial design portion	-1	-1	-1	0.02
	1	-1	-1	0.14
	-1	1	-1	0.22
	1	1	-1	0.31
	-1	-1	1	0.5
	1	-1	1	0.66
	-1	1	1	0.55
	1	1	1	0.65
Axial points	-1.682	0	0	0.57
	1.682	0	0	0.58
	0	-1.682	0	0.13
	0	1.682	0	0.62
	0	0	-1.682	0.54
	0	0	1.682	0.74
Center points	0	0	0	0.08
	0	0	0	0.04
	0	0	0	0.11
	0	0	0	0.14
	0	0	0	0.09
	0	0	0	0.13

Example (cont'd): Data for Regression Analysis

- Design matrix and measurement vector

X =

1	-1	-1	-1	1	1	1	1	1	1
1	1	-1	-1	1	1	1	-1	-1	1
1	-1	1	-1	1	1	1	-1	1	-1
1	1	1	-1	1	1	1	1	-1	-1
1	-1	-1	1	1	1	1	1	-1	-1
1	1	-1	1	1	1	1	-1	1	-1
1	-1	1	1	1	1	1	-1	-1	1
1	1	1	1	1	1	1	1	1	1
1	-1.682	0	0	2.828	0	0	0	0	0
1	1.682	0	0	2.828	0	0	0	0	0
1		-1.682	0	0	2.828	0	0	0	0
1	0	1.682	0	0	2.828	0	0	0	0
1	0	0	-1.682	0	0	2.828	0	0	0
1	0	0	1.682	0	0	2.828	0	0	0
1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0

y =

0:02
0:14
0:22
0:31
0:50
0.05
0:55
0.05
0:57
0:58
0:13
0.04
0:54
0.05
0:08
0:04
0:11
0:14
0:09
0:13

Example (cont'd): Least-squares Regression Analysis

- Estimate of coefficient vector:

$$b = \left(X^T X\right)^{-1} X^T y = \begin{bmatrix} 0.1040 \\ 0.0356 \\ 0.0904 \\ 0.1469 \\ 0.1316 \\ 0.0609 \\ 0.1546 \\ -0.0113 \\ 0.0063 \\ -0.0413 \end{bmatrix}$$

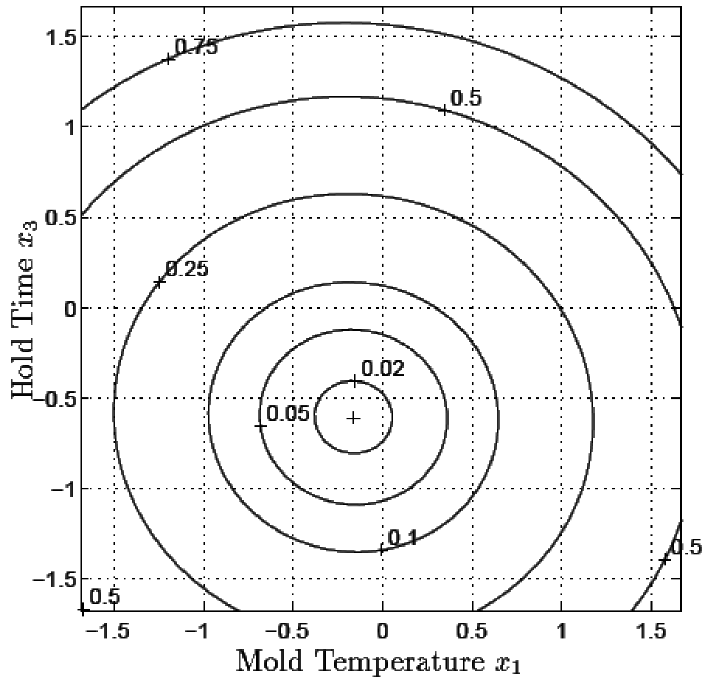
- Response surface model:

$$\begin{aligned} y = {} & 0.1040 + 0.0356x_1 + 0.0904x_2 + 0.1469x_3 + 0.1316x_1^2 + 0.0609x_2^2 \\ & + 0.1546x_3^2 - 0.0113x_1x_2 + 0.0063x_1x_3 - 0.0413x_2x_3 \end{aligned}$$

- Coefficients involving x_2 small → its effect ignored and x_2 fixed at -1

Example (cont'd): Response Surface

- Contour plot of response surface $\hat{y}$ versus mold temperature and holding time when screw speed x_2 = -1
- Lowering mold temperature and using shorter holding time can reduce y from a current value of ~ 0.1 to as low as ~ 0.01 → ten-fold reduction



Statistical Properties of the Least-squares Model

- Response estimate is *unbiased*: prediction is correct on the average

$$E\left(\hat{\eta}\left(x\right)\right) = \eta\left(x\right)$$

- Prediction-error covariance matrix is a function of *x* (the uncertainty in prediction depends on the point x at which the prediction is made)

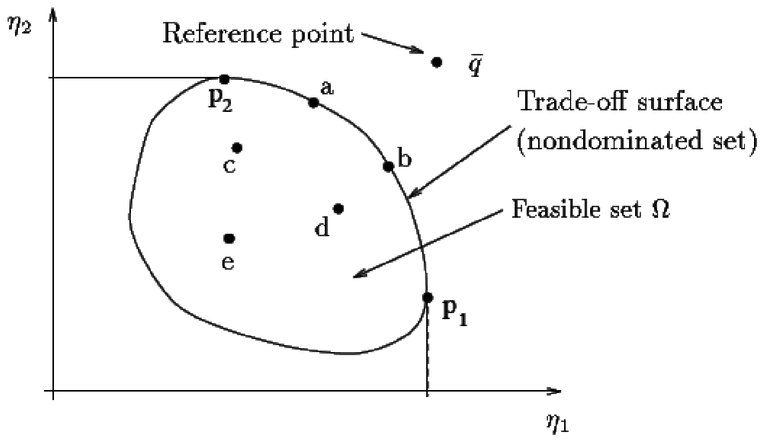
$$Cov\left(\hat{\eta}\left(x\right)\right) = z^T\left(x\right)\left(X^T X\right)^{-1} z\left(x\right)\sigma^2$$

σ is the standard deviation of the additive noise

- Model optimum versus true optimum
  - In general $E\left[\arg\max_x \hat{\eta}\left(x\right)\right] \neq \arg\max_x \hat{\eta}\left(x\right)$
  - *Confidence regions* may be obtained via simulations for the optimum design point *x**

4. Multiobjective Design for Robustness

- Multiobjective Optimization/Vector Optimization: a mulitobjective optimum is any solution in a set called the non-dominated (or Pareto optimal) set
  - Nondominated solution ⇒ no other solution is better with respect to all objectives
  - Choice of any particular nondominated solution based on user's preferences
- Illustration: Case of maximizing two objectives
  - *a*, *b*, p_1, p_2 nondominated
  - *a* dominates *c* and *e*
  - *b* dominates *d* and *e*



Approaches to Multiobjective Optimization

- Two groups of approaches for incorporating a decision-maker's preferences to choose the non-dominated solution
  - **Indirect methods**: based on utility functions
  - **Direct methods**: greater interaction possible
- **Direct methods:**
  - **Goal programming methods**
    - Establish a goal level of achievement for each objective, and find a feasible solution in the parameter space that achieves the goals
    - If goal is not reachable, get as close as possible.
  - **Reference-point method**
    - Decision-maker's preferences expressed as a reference-point in the objectives space (similar to goal vector)
    - Solution obtained by maximizing a scalarizing function — guarantees a non-dominated solution whether or not the reference-point is achievable

Goal Programming

- Generic formulation

$$goal_j\left(\eta_j(x)=t_j\right);\ j=1,\cdots,M$$
$$s.t.\ x\in\Xi$$

- Archimedian GP

$$\min_{x\in\Xi}\left\|W\left(\eta(x)-t\right)\right\|_p, \qquad W=diag\left[w_1,w_2,\cdots,w_M\right],\ w_j>0$$

  - Essentially a (non) linear programming problem
  - Nonlinear objectives: use L_2 metric; linear objectives: use L_1 metric

$$\min\sum_{j=1}^{M}w_j\left(d_j^-+d_j^+\right)$$
$$s.t.\ \eta_j(x)+d_j^--d_j^+=t_j;\ \ j=1,\cdots,M$$
$$x\in\Xi$$
$$d_j^-\geq 0;\ d_j^+\geq 0;\quad j=1,\cdots,M$$

d_j^-: negative deviation (under-achievement) of j-th objective goal t_j:

d_j^+: positive deviation (over-achievement).

Goal Programming

- Preemptive GP (Lexicographic GP)

$$lex \min_{x \in \Xi} \left\{ \left\| \eta_1(x) - t_1 \right\|_p, \left\| \eta_2(x) - t_2 \right\|_p, \cdots \left\| \eta_M(x) - t_M \right\|_p \right\}$$

- Iterative procedure

1. $\min_{x \in \Xi} \left\| \eta_1(x) - t_1 \right\|_p$;

2. for $j = 2, \cdots, M$,

do

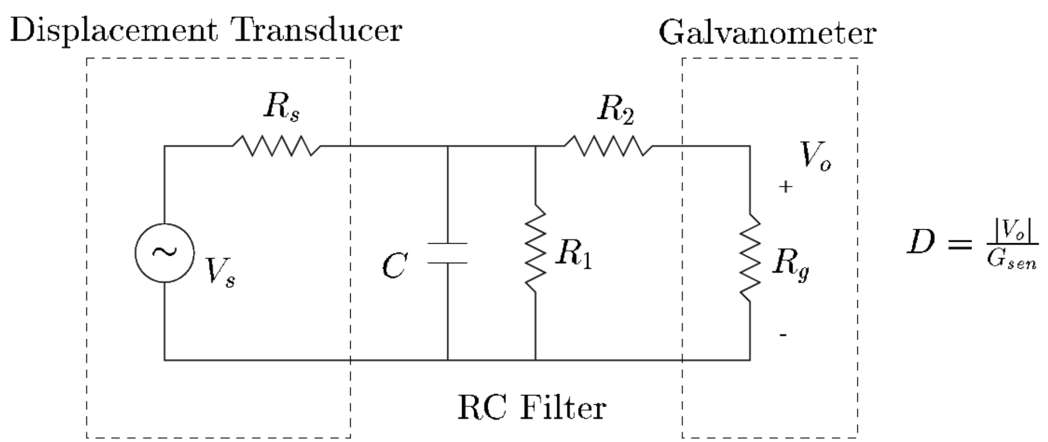
$$\min_{x \in \Xi} \left\| \eta_j(x) - t_j \right\|_p$$

$$s.t. \left\| \eta_k(x) - t_k \right\|_p \leq d_k^* + \Delta_k; \quad k = 1, \cdots, j-1$$

end do

- d_k^*'s: optimal values of $\left\| \eta_k(x) - t_k \right\|_p$ in each step k
- $\Delta_k \geq 0$: relaxation parameter for $\eta_k(x)$ (set to zero or chosen by DM)

Example: Passive Filter Design



- Two responses
  - Cut-off frequency

$$f_c = \frac{(R_2 + R_g)(R_s + R_1) + R_1 R_s}{2\pi (R_2 + R_g) R_1 R_s C}$$

  - Full-scale galvanometer deflection

$$D = \frac{|V_0|}{G_{sen}} = \frac{|V_s| R_g R_1}{G_{sen}\left[(R_2 + R_g)(R_s + R_1) + R_1 R_s\right]}$$

  - All circuit parameters assumed normally distributed about their nominal values

Example: Passive Filter Design

- GP problem:

$$\text{goal}\left(\bar{f}_C = 6.84Hz.\right)$$

$$\text{goal}\left(\bar{D} = 3.0\text{in.}\right)$$

$$s.t.\ \bar{f}_C = \frac{\left(\bar{R}_2 + \bar{R}_g\right)\left(\bar{R}_s + \bar{R}_1\right) + \bar{R}_1\bar{R}_s}{2\pi\left(\bar{R}_2 + \bar{R}_g\right)\bar{R}_1\bar{R}_s\bar{C}}$$

$$\bar{D} = \frac{\left|\bar{V}_s\right|\bar{R}_g\bar{R}_1}{\bar{G}_{sen}\left[\left(\bar{R}_2 + \bar{R}_g\right)\left(\bar{R}_s + \bar{R}_1\right) + \bar{R}_1\bar{R}_s\right]}$$

$$\bar{R}_2 > 0, \bar{C} > 0,$$

  - Ignored bias in first-stage GP
  - R_1 taken as 'independent' factor; R_2 and C computed from

$$R_2 = \left(\frac{|V_s|R_g - DG_{sen}R_s}{DG_{sen}}\right)\frac{R_1}{R_s + R_1} - R_g,$$

$$C = \frac{|V_s|R_g}{2\pi f_c R_s\left(|V_s|R_g - DG_{sen}R_s\right)}\left(\frac{R_s + R_1}{R_1}\right).$$

- Second-stage (Archimedian) GP

$$\min_{R_1}\left\{w_1\left(f_c - 0.84\right)^2 + w_2\left(d - 3.0\right)^2 + w_3\sigma_{f_c}^2 + w_4\sigma_D^2\right\}$$

Example: Passive Filter Design

$\bar{R}_1(\Omega)$	$\bar{R}_2(\Omega)$	$\bar{C}(\mu F)$	$\bar{f}(Hz)$	$\sigma_{fc}(Hz)$	$\bar{D}(in)$	$\sigma_D(in)$	Cost
150	249.31	416.02	6.8464	0.1842	3.0002	0.0533	0.0368
200	292.72	369.8	6.8459	0.1811	3.0003	0.053	0.0356
250	324.4	342.06	6.8459	0.1791	3.0005	0.0526	0.0349
300	348.54	323.57	6.8455	0.1778	3.0004	0.0525	0.0344
350	367.54	310.36	6.8458	0.1768	3.0005	0.0524	0.034
400	382.89	300.46	6.8456	0.1759	3.0005	0.0524	0.0337
450	395.54	292.75	6.8457	0.1755	3.0007	0.0523	0.0336
500	406.16	286.59	6.8452	0.175	3.0007	0.0524	0.0334
550	415.19	281.55	6.8453	0.1747	3.0007	0.0522	0.0333
600	422.96	277.35	6.8457	0.1745	3.0007	0.0524	0.0332
650	429.73	273.79	6.8451	0.1745	3.0008	0.0524	0.0332
1000	460.18	258.86	6.8449	0.1737	3.0008	0.0525	0.033
1100	465.67	256.34	6.8448	0.1736	3.0007	0.0525	**0.0329**
1200	470.32	254.23	6.8454	0.1733	3.0008	0.0526	**0.0328**
1300	474.33	252.46	6.8449	0.1738	3.0007	0.0527	0.033
1400	477.8	250.93	6.8451	0.1735	3.0007	0.0527	0.0329
1500	480.85	249.61	6.845	0.1735	3.0008	0.0527	0.0329
1600	483.54	248.46	6.8448	0.1734	3.0008	0.0528	**0.0329**
1700	485.94	247.44	6.8449	0.1733	3.0008	0.0527	0.0328
1800	488.08	246.53	6.8454	0.1733	3.0007	0.0527	0.0328
1900	490.02	245.72	6.8448	0.1735	3.0008	0.0527	0.0329
2000	491.77	244.99	6.8448	0.1735	3.0008	0.0528	0.0329

Remarks

- R_1 >1000 , and R_2 and C obtained from R_1, results in nearly optimal design
- For the chosen equal weights, σ_{fc} makes the most significant contribution to the cost function ⇒ justified in ignoring the biases in $\bar{f}_c$ *and* $\bar{D}$ from their targets
- Problem with GP: if all goals are achievable, solution is not non-dominated ⇒ under-achievement.

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Summary

- Measures of Quality Loss
- Robust Parameter Design (Taguchi)
- Response Surface Methods
- Multi-objective optimization