Dept. of ECE **Spring 2013**
Univ. of Conn. **KRP**

<u>Homework Set # 1 (Due February 28, 2013)</u>

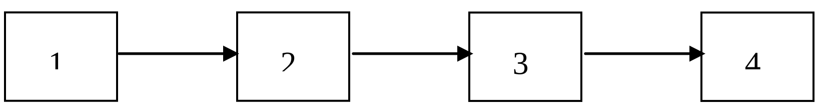
1. Little’s theorem is valid for any work conserving queuing discipline.

   a) Derive Little’s theorem relating the average response time and average system queue length assuming a last-come first-served (LCFS) queuing discipline.
   b) Show that Little’s theorem is valid for systems where the order of customer service arbitrary, and where customer service can be interrupted to serve customers of higher priority.

   Hint: You may want to refer to the following *classic* papers on Little’s formula.

   i) Little, J.D.C., “A Proof of the Queuing Formula $L = \lambda W$," <u>Operations Research</u>, Vol. 9, 1961, pp. 383-387.
   ii) Jewell, W.S., “A Simple Proof of $L = \lambda W$," <u>Operations Research</u>, Vol. 15, 1967, pp. 1109-1116.
   iii) Eilon, S., “A Simpler Proof of $L = \lambda W$," <u>Operations Research</u>, Vol. 17, 1969, pp. 915-916.
   iv) Stidham, S., Jr., “A Last Word on $L = \lambda W$," <u>Operations Research</u>, Vol. 22, 1974, pp. 417-421.

2. Four machines are arranged in an assembly line to execute a stream of jobs.



Four Stage Processor Pipeline

Six jobs are executed on this system, and require the following processing time at each stage (measured in hours):

a) Construct a schedule for executing the six jobs on this assembly.
b) Compute the average response time for the above schedule:

Table I: Processing Times per Stage

Job	Stage 1	Stage 2	Stage 3	Stage 4
1	4	4	5	4
2	2	5	8	2
3	3	6	7	4
4	1	7	5	3
5	4	4	5	3
6	2	5	5	1

$$R = \frac{1}{N}\sum_{k=1}^{N} R_k;\ N = 6$$

c) Let Q*(t)* denote the number of jobs in the system, either waiting to be executed or in execution, at time *t*. Calculate the average number of jobs in the system, Q:

$$Q = \frac{1}{T_F} \int_0^{T_F} Q(t)dt$$

d) Show that

$$Q = \frac{N}{T_F} R = XR;\ X = Average\,Throughput$$

3. Let X *and* Y be independent exponential random variables, each with mean $1/\mu$.. Further, let $Z = \min(X, Y)$, *and* $W = \max(X, Y)$. Find *E(Z), E(W), Var(Z) and Var(W).*

4. Problem 2, Chapter 8, Page 303.

5. Problem 6, Chapter 8, Page 304.

6. Problem 10, Chapter 8, Page 305.