Lectures 11

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EE 336
Stochastic Models for the Analysis of Computer Systems And Communication Networks

Outline

- Random Access Networks
- Pure and Slotted Aloha
- Stability Issues
- Stabilization of Slotted Aloha
- Splitting (Tree) Algorithms
- Markov Chain Analysis
- Summary

Random Access Concepts - 1

Basic idea:

- Have a set of nodes or users
- Each node has a queue of packets to be transmitted
- The channel is the common server (e.g., satellite, multi-drop telephone line, multi-tap bus (Ethernet), packet radio)
- The server does not know which node contains packets. Similarly, nodes are unaware of packets at other nodes so that the knowledge of the state of the system is *distributed*.

Two extremes:

1) "Free-for-all" or totally distributed approach. Each node sends its packets whenever it gets them.

   Problem: Two or more nodes may decide to transmit at almost the same time so that their signals overlap on the channel, and are garbled. Such an overlap of signals is called *collision*. Good idea under light load conditions.

2) "Perfectly scheduled" or centralized approach. Each node is asked to transmit packets, if any, at specified time slots, e.g., TDM.

   Problem: Inefficient channel use under light load conditions.

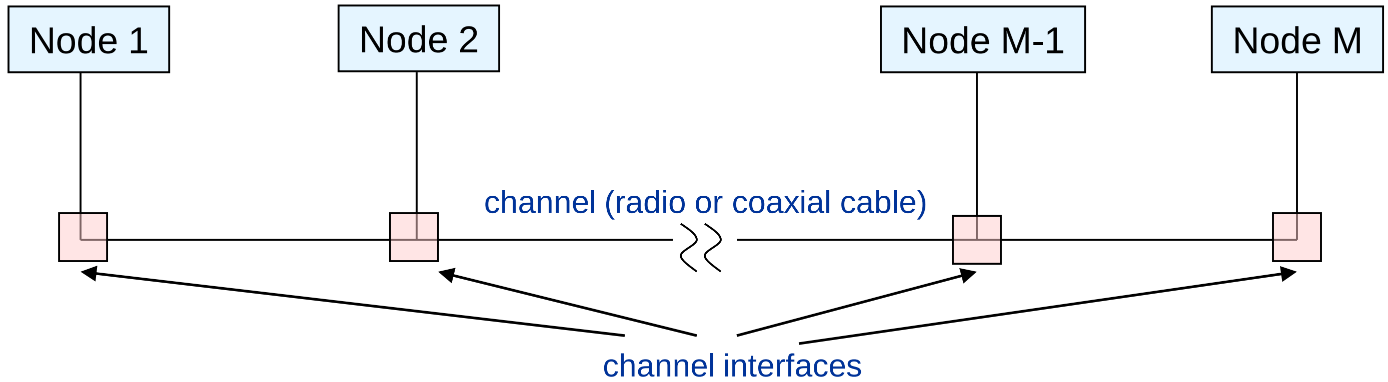
Random Access Concepts - 2

- The first free-for-all approach was developed for long radio links and for satellite communications: *Pure Aloha* at the univ. of Hawaii
  Slotted Aloha to improve the performance of pure Aloha
- When nodes are *close* together, the propagation delay is small. In these cases, a node can "listen" to the channel to determine if it is busy before attempting a transmission. If the channel is sensed busy, the node can defer its transmission until the channel is sensed to be idle. This process is called "carrier sensing" and the corresponding scheme is called Carrier Sense Multiple Access (CSMA) or Listen Before Talk (LBT). CSMA is useless for satellite channels, since the propagation delays >> packet transmission times. For small propagation delay networks (e.g., LANs), CSMA-type protocols can provide significantly smaller average delays and higher throughputs than the Aloha type methods (propagation delay ≈ 5 μs/km)
- In local area networks, you can do one more thing: a node can listen while transmitting. If an interfering signal is detected, transmission can be aborted immediately. This results in Carrier-Sense Multiple Access / Collision Detect (CSMA/CD) protocol.

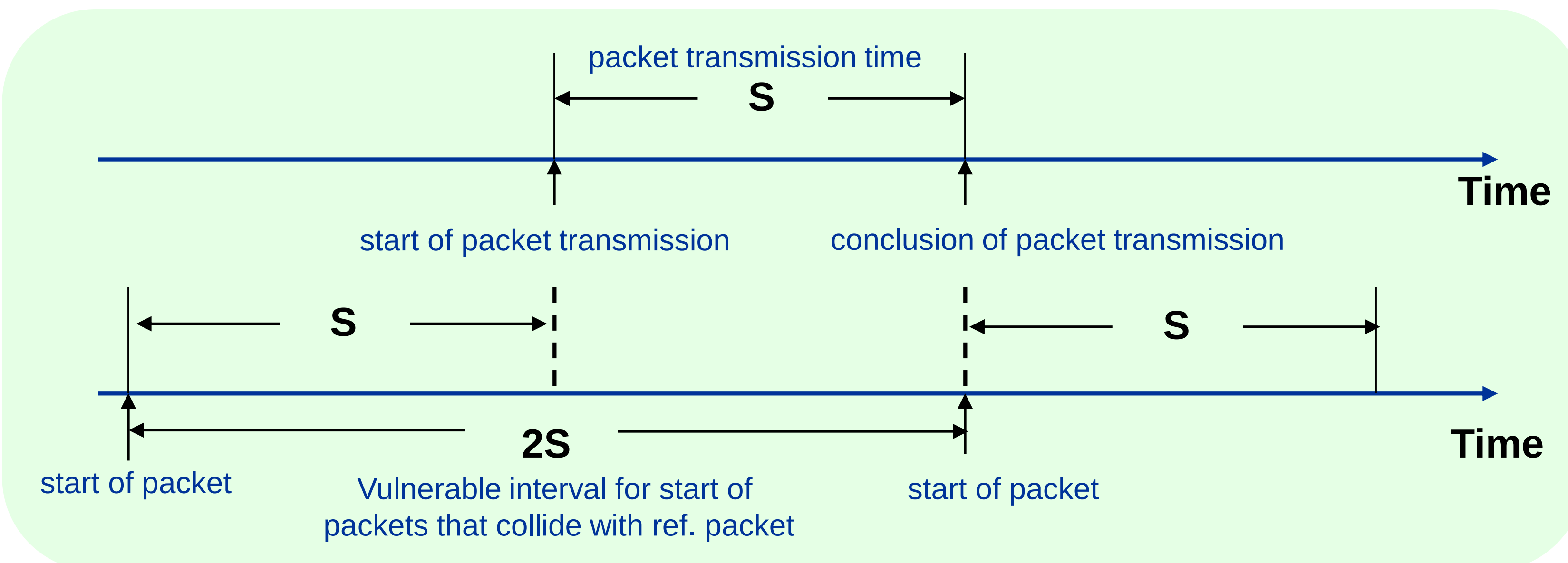
So we have an interesting array of random access schemes:

Pure Aloha ←	focus of Lecs.11-12	→ Slotted Aloha
CSMA	focus of Lec.13	CSMA/CD

Pure Aloha Analysis - 1



Consider a packet transmission from a reference node, say node 1.



Pure Aloha Analysis - 2

Let λ denote the arrival rate of packets (packets / second)

S packet transmission time

ρ = load offered to the communication channel

= average number of successful transmissions per packet transmission time, S

= throughput

Λ = new and retransmitted messages per second

= attempted packet transmissions per second = offered traffic

$G = \Lambda S$ = attempted packet transmissions per packet transmission time S = offered load

Assumptions: each node holds no more than one packet

constant length

noise-free channel

node transmits a packet before another arrives

G is Poisson

Pure Aloha Analysis - 3

By definition, throughput $= pr\{\text{successful transmission}\} \cdot \text{offered load}$

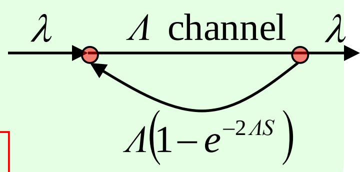
$$\rho = G \cdot pr\{\text{successful transmission}\}$$

$$= G \cdot pr\{0 \text{ arrives in an interval of length } 2S\}$$

$$= G \cdot e^{-2\Lambda S}$$

$$\boxed{\rho = G \cdot e^{-2G}}$$

Alternatively, $\Lambda = \lambda + \Lambda \cdot pr\{\text{collision}\}$



$$= \lambda + \Lambda \cdot (1 - e^{-2\Lambda S})$$

$$\lambda = \Lambda \cdot e^{-2\Lambda S}$$

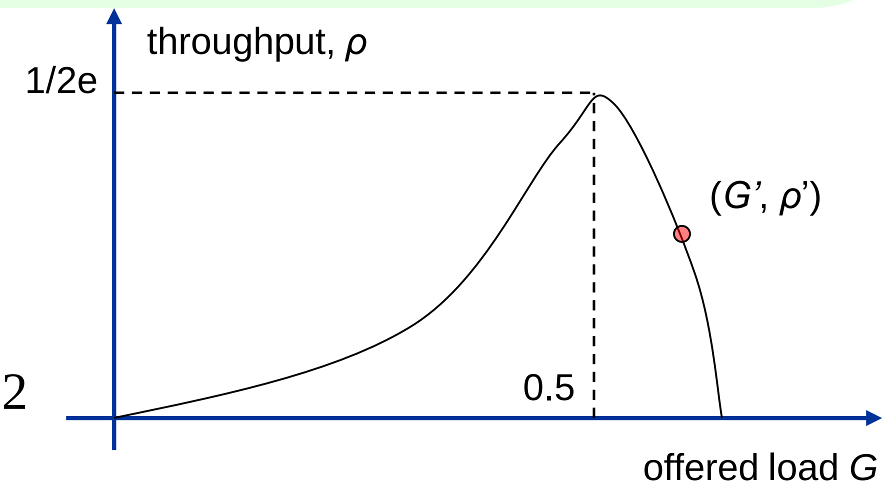
$$\boxed{\rho = G \cdot e^{-2G}}$$

Max at: $\dfrac{d\rho}{dG} = 0$

$$\Rightarrow e^{-2G} - 2Ge^{-2G} = 0$$

$$G = 1/2$$

$$\rho = 1/2e = .184 \quad \text{at} \quad G = 1/2$$



Pure Aloha Analysis - 4

- An important assumption in deriving the throughput equation is the assumption of steady state. However, this assumption may not be true for *G>0.5*.

$$\boxed{\begin{array}{l} G\uparrow \;\Rightarrow\; \rho\downarrow \;\Rightarrow\; \text{more collisions} \;\Rightarrow\; G\uparrow \;\Rightarrow\; \rho\downarrow \;\&\; \text{eventrually} \\ G\rightarrow\infty \;\&\; \rho\rightarrow 0 \;\; \text{and the channel is said to be saturated.} \end{array}}$$

- **Delay analysis:**

  For each packet, the average number of attempts before successful transmission is given by

$$\boxed{\frac{G}{\rho} = e^{2G}}$$

- Average # of unsuccessful attempts per successfully transmitted packet:

$$\boxed{\frac{G}{\rho} - 1 = e^{2G} - 1}$$

Pure Aloha Analysis - 5

- **Now, what do we do when there is a collision?:**

Each node reschedules its colliding packet at some randomly chosen future time. This rescheduling causes a delay during which the packet is said to be in a state of “backoff”. Suppose that the average backoff delay is B, then, the delay (response time) per packet is:

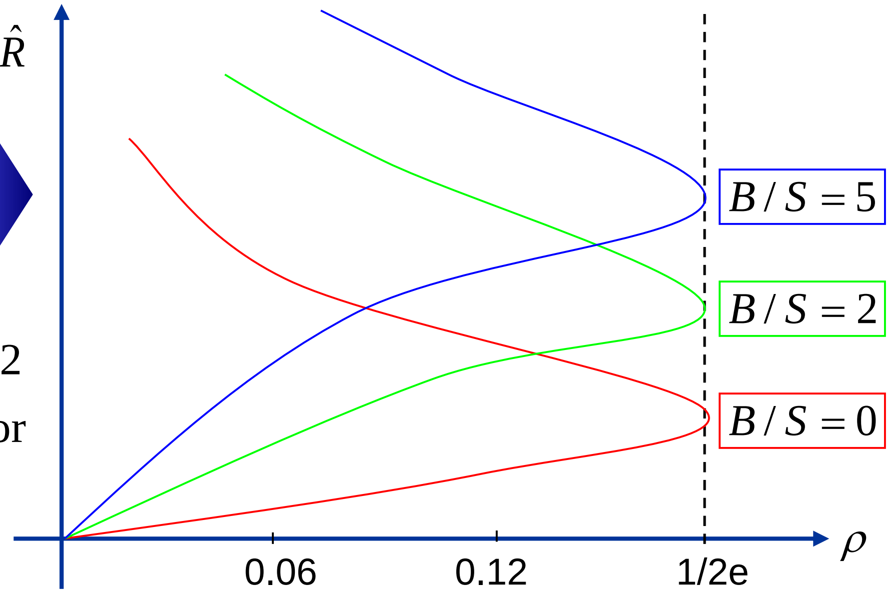
$$\boxed{R = S + (e^{2G} - 1)(S + B)}$$

- Normalized delay is:

$$\boxed{\hat{R} = \frac{R}{S} = 1 + (e^{2G} - 1)\left(1 + \frac{B}{S}\right) = 1 + \left(\frac{G - \rho}{\rho}\right)\left(1 + \frac{B}{S}\right)}$$

$$\boxed{B = 0 \Rightarrow \hat{R}_{\min} = e^{2G}}$$

Since G is a function of ρ (Recall $\rho = Ge^{-2G}$), we can plot $\hat{R}$ vs. ρ.



$\hat{R}_{\min} = 2.718 = e$ at $\rho = 1/2e$ or $G = 1/2$

Since for $B = 0$, the system is unstable for $G > 0.5$, $\hat{R}_{\min} \to \infty$ ($\Rightarrow$ No steady-state)

Comparison with TDMA - 1

- **Idealized scheme:**

$$\text{arrived rate from each node} = \lambda$$

$$\text{suppose } m \text{ users} \Rightarrow \lambda_T = m\lambda$$

$$\rho = \lambda_T S = m\lambda S = \text{fraction of time channel transmits good packets}$$

- **M/D/1 queue: single user case**

$$\boxed{R_I = S + S \cdot \frac{\rho}{2(1-\rho)}}$$

$$\boxed{\hat{R}_I = 1 + \frac{\rho}{2(1-\rho)} = \frac{2-\rho}{2(1-\rho)}}$$

- **TDM: m user case**

$$\boxed{R_{TDMA} = S + \frac{m}{2}S + \frac{m\rho S}{2(1-\rho)}}$$

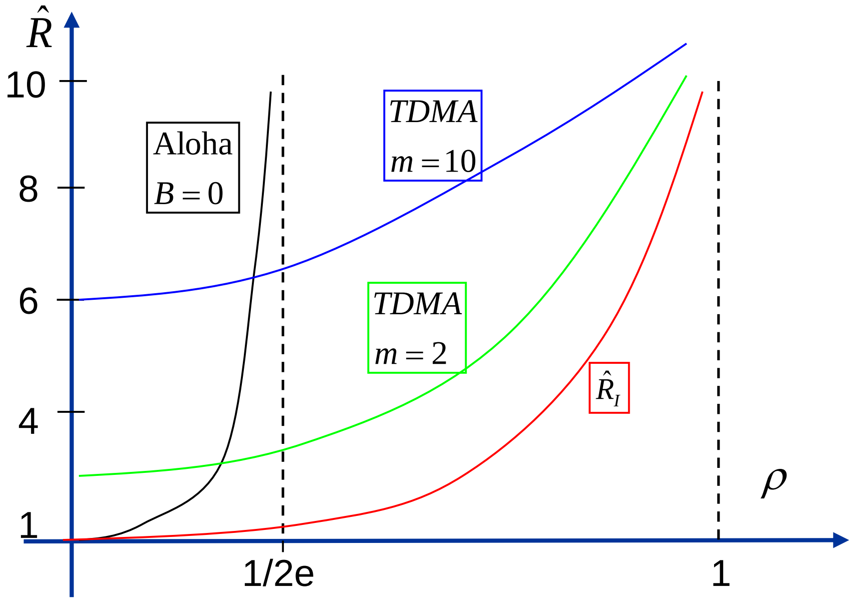
transmission time

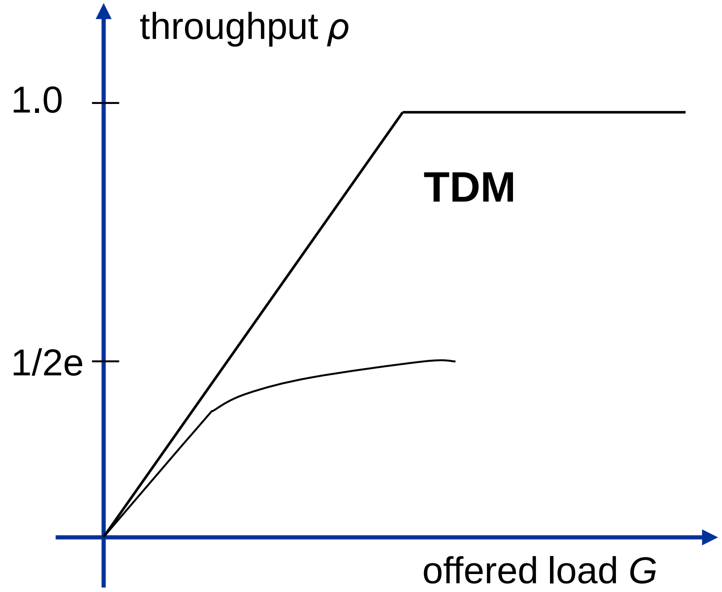
slot synchronization delay

waiting time

$$\boxed{\hat{R}_{TDMA} = 1 + \frac{m}{2} + \frac{m\rho}{2(1-\rho)} = 1 + \frac{m}{2(1-\rho)}}$$

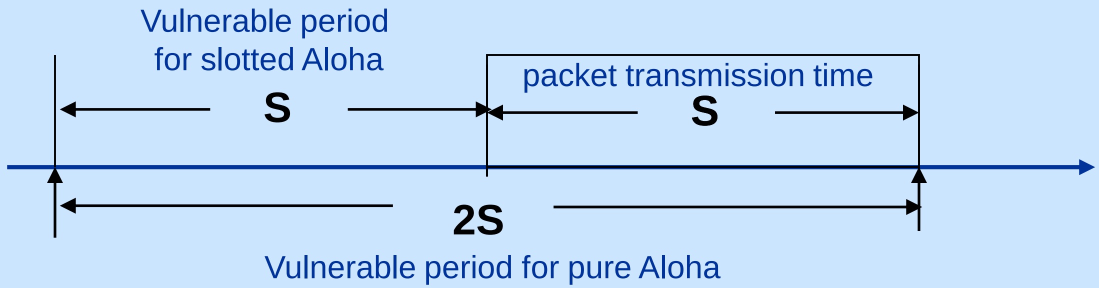
Comparison with TDMA- 2





Slotted Aloha - 1

- Time is divided into segments of fixed length, S = the packet transmission time
  → all packets must have the same length.
- If a packet arrives during a slot, it must be delayed until the beginning of the next slot.



- **Let :** $\Lambda = \text{arrival rate} + \text{retransmitted rate}$

$$\lambda = \text{arrival rate}$$

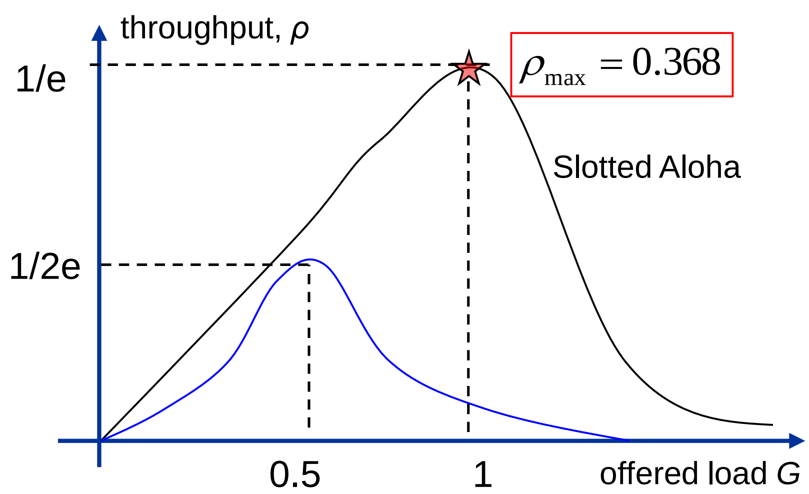
$$\Lambda = \lambda + \Lambda \cdot pr\{\text{collision}\}$$

$$pr\{\text{collision}\} = 1 - e^{-\Lambda S}$$

$$\Rightarrow \Lambda = \lambda + \Lambda \cdot \left(1 - e^{-\Lambda S}\right)$$

$$\Lambda S = G = \rho + G \cdot \left(1 - e^{-G}\right)$$

or $\boxed{\rho = G \cdot e^{-G}}$



Slotted Aloha - 2

Alternate Analysis :

Let $m = \#$ of nodes

$\rho_i = prob\{\text{node } i \text{ successfully transmits a packet in a slot}\}$

$1\text{-}\rho_i = prob\{\text{node } i \text{ does not successfully transmit a packet in a slot}\}$

$G_i = prob\{\text{node } i \text{ attempts a transmission in a slot}\}$

clearly, $\rho_i \le G_i$

$$\boxed{\rho_j = G_j \prod_{i=1, i\neq j}^{m} (1 - G_i)}$$

Assume identical stations $\ni$ $\rho_i = \dfrac{\rho}{m};\ G_i = \dfrac{G}{m}$

then $$\boxed{\rho = G \prod_{i=1, i\neq j}^{m} \left(1 - \frac{G}{m}\right) = G\left(1 - \frac{G}{m}\right)^{m-1}}$$

Now let $m \to \infty$, then $\lim_{m\to\infty}\left(1 - \dfrac{G}{m}\right)^{m-1} = e^{-G}$, $\rho = Ge^{-G}$

$$\frac{d\rho}{dG} = 0 \Rightarrow \left(1 - \frac{G}{m}\right)^{m-1} + (m-1)G\left(1 - \frac{G}{m}\right)^{m-2}\frac{-1}{m} = 0$$

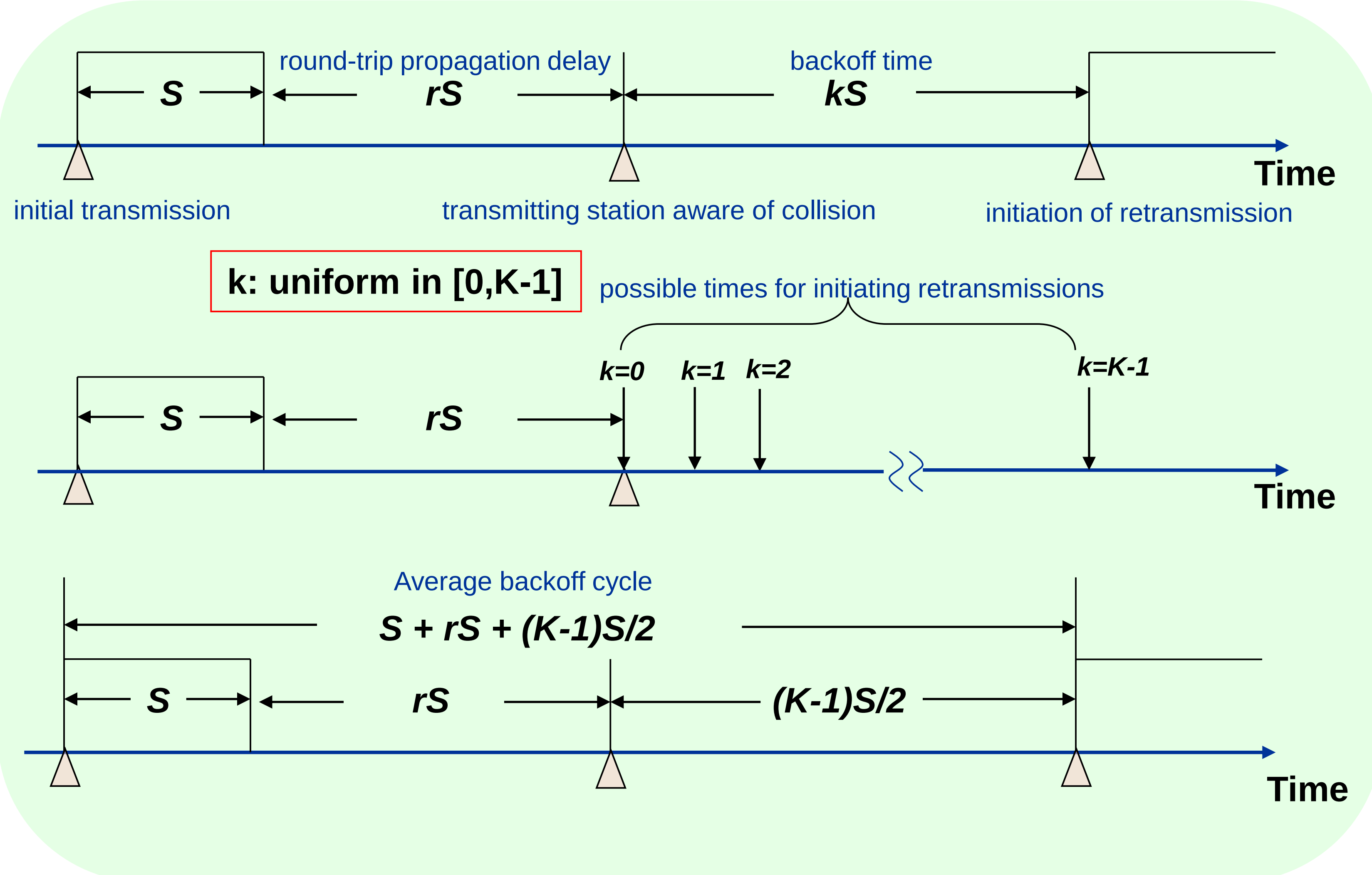
$$\Rightarrow \left(1 - \frac{G}{m}\right) - \left(\frac{m-1}{m}\right)G = 0$$

$$\Rightarrow \boxed{G = 1,\quad \rho_{\max} = \left(1 - \frac{1}{m}\right)^{m-1}}$$

m	1	2	5	10	20	100	∞
$\rho_{\max}$	1	0.5	0.410	0.387	0.377	0.370	0.368

For $m > 20$, can assume asymptotic approximation.

Delay Analysis - 1



Delay Analysis - 2

- **Packet delay consists of:**
  1) Waiting time after arrival until the beginning of the next slot;
  2) The delay due to retransmissions;
  3) The packet transmission time; and
  4) The propagation delay

1) Residual time: $\frac{S^2}{2S} = \frac{S}{2}$

2) Retransmission delay $=$ *Av. # of retransmissions* $\times$ *Av. length of retransmission (backoff) cycle*

$$= H \cdot \left(r + \frac{K+1}{2} \right) \cdot S$$

3) Packet transmission time $= S$

4) If nodes are uniformly distributed and the end-to-end delay is τ *seconds, the propagation delay corresponds to residual time of uniform distribution* $= \frac{\tau}{3}$

so, $\boxed{R = S + \frac{S}{2} + \frac{\tau}{3} + H \cdot \left(r + \frac{K+1}{2} \right) \cdot S}$

Normalized delay:

$$\boxed{\hat{R} = \frac{R}{S} = \frac{3}{2} + \frac{a}{3} + H \cdot \left(r + \frac{K}{2} + 0.5 \right)}$$

where, $a = \frac{\tau}{S}$ *normalized end-to-end propagation delay* $\approx 0.01, \; r \approx 1$

Delay Analysis - 3

Computation of H :

Let q_a = prob. of a successful transmission, given that the transmission is a new packet

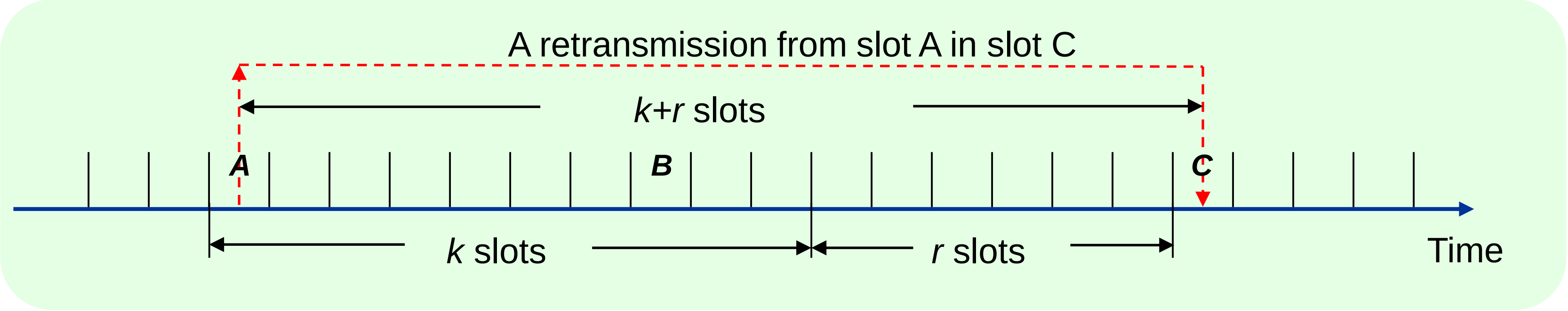
q_r = prob. of a successful transmission, given that the transmission is a retransmission

Let $p_i = prob\,\{\text{a packet requires exactly } i \text{ retransmissions}\}$

$$= (1-q_a)(1-q_r)^{i-1} \cdot q_r \qquad i \geq 1$$

$$\boxed{H = \sum_{i=1}^{\infty} i p_i = \frac{1-q_a}{q_r}}$$

Computation of q_a and q_r :



$$q_a = prob\{\text{successful transmission in slot } C \mid \text{packet is a new arrival}\}$$

$$q_r = prob\{\text{successful transmission in slot } C \mid \text{packet is a retransmission}\}$$

Delay Analysis - 4

- **Computation of q_a and q_r (cont.):**

If the successfully transmitted packet is a retransmission, we need:

- No other packets that collided in slot *A* should be retransmitted in slot *C*. Let q_c be this probability.
- No new packets should be generated in slot *C*. ➡ prob of the event = $e^{-\rho}$.
- No packets that collided in one of the (*k*-1) slots, other than *A*, should be rescheduled for slot *C*. The prob of no retransmission in a specific slot, other than *A*, taking place in slot *C* is q_0. Then the prob. Of this event is: q_0^{k-1}

- Assuming independence:

$$\boxed{q_r = q_c \cdot e^{-\rho} \cdot q_0^{k-1}}$$

- Similarly, q_a corresponds to the following two events:
  1) No other new arrivals are generated in the current slot *C* ; and
  2) No retransmissions occur in slot *C* from collisions in earlier slots.

$$\boxed{q_a = e^{-\rho} \cdot q_0^{k}}$$

Delay Analysis - 5

Computation of q_a and q_r (cont.):

Determination of q_c : *prob*{one or more packets is transmitted in slot A (in addition to the one successfully transmitted in slot C) and none of these additional packets is retransmitted in slot C | collision in slot A}

Let X be the event without conditions on what happed in slot A.

$$q(j) = prob\left\{event\ X,\ j\ additional\ packets\ transmitted\ in\ slot\ A\right\}$$

$$= prob\left\{j\ arrivals\ in\ slot\ A\right\} \cdot prob\left\{not\ retransmitting\ a\ particular\ collided\ packet\ in\ slot\ C\right\}$$

$$= \frac{G^j e^{-G}}{j\,!}\left(1-\frac{1}{K}\right)^j$$

$$q_c = prob\left\{event\ X \,\middle|\, a\ collision\ occurs\ in\ slot\ A\right\}$$

$$= \frac{\sum_{j=1}^{\infty} q(j)}{1-e^{-G}} = \sum_{j=1}^{\infty}\frac{e^{-G}}{1-e^{-G}}\frac{\left[G\left(1-\frac{1}{K}\right)\right]^j}{j\,!}$$

$$= \frac{e^{G\left(1-\frac{1}{K}\right)}-1}{1-e^{-G}}\cdot e^{-G} = \frac{e^{-G/K}-e^{-G}}{1-e^{-G}}$$

Note that as $K \to \infty,\ q_c \to 1.$

Delay Analysis - 6

- Determination of q_0 : *prob*{no transmissions from a collision in a slot other than slot A (for example, slot B) appear in slot A}. This event can occur in three mutually exclusive ways:

1) No transmission at all occurs in slot B $\Rightarrow \; e^{-G}$

2) A successful transmission occurs in slot B, and therefore no retransmission is needed $\Rightarrow Ge^{-G}$

3) Two or more transmissions take place in slot B, but none is retransmitted in slot C $= \sum_{j=2}^{\infty} q(j)$

$$\therefore \; q_0 = e^{-G} + Ge^{-G} + \sum_{j=2}^{\infty} q(j)$$

$$= e^{-G} + Ge^{-G} + \sum_{j=2}^{\infty} \frac{G^j e^{-G}}{j\,!}\left(1-\frac{1}{K}\right)^j$$

$$= e^{-G} + Ge^{-G} + e^{-G}\left[e^{G\left(1-\frac{1}{K}\right)} - 1 - G\left(1-\frac{1}{K}\right)\right]$$

$$\boxed{q_0 = e^{-G/K} + \frac{G}{K}e^{-G}}$$

$q_0 \to 1 \quad as \quad K \to \infty$

Also note that $q_0^K \to e^{\rho - G} \quad as \; K \to \infty$

So, $\boxed{q_r = \left(\frac{e^{-G/K} - e^{-G}}{1-e^{-G}}\right)\left(e^{-G/K} + \frac{G}{K}e^{-G}\right)^{K-1} e^{-\rho}}$; $\boxed{q_a = \left(e^{-G/K} + \frac{G}{K}e^{-G}\right)^{K} e^{-\rho}}$

Delay Analysis - 7

So, $\hat{R} = \frac{3}{2} + \frac{a}{3} + \frac{1-q_a}{q_r}\left(r + \frac{K}{2} + \frac{1}{2}\right)$

as $K \to \infty$, $q_a \to e^{-G}$ *and* $q_r \to e^{-G}$ *so that*

$$\boxed{\hat{R} = \frac{3}{2} + \frac{a}{3} + \frac{1-e^{-G}}{e^{-G}}\left(r + \frac{K}{2} + \frac{1}{2}\right)}$$

since prob{successful transmission} $= \frac{\rho}{G}$

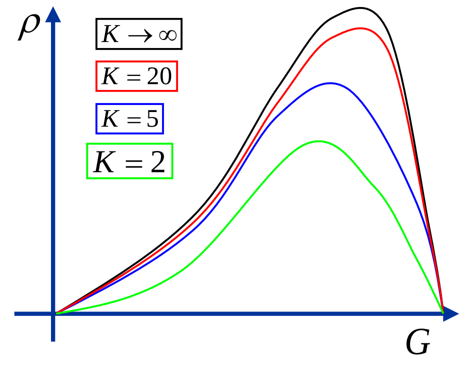
Av. # of times a packet must be transmitted until success:

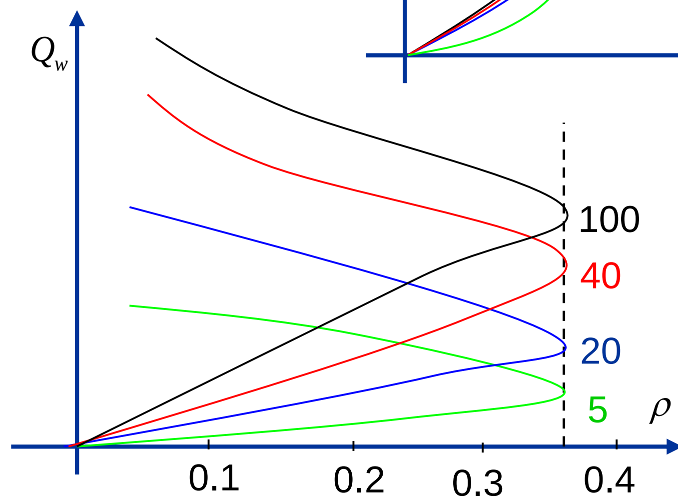
$$1 + \left(1 - \frac{\rho}{G}\right) + \left(1 - \frac{\rho}{G}\right)^2 + \cdots = \frac{G}{\rho}$$

$$\therefore \quad 1 + H = \frac{G}{\rho} \quad \Rightarrow \quad 1 + \frac{1-q_a}{q_r} = \frac{G}{\rho}$$

or, $\boxed{\rho = G \cdot \frac{q_r}{1 + q_r - q_a}}$

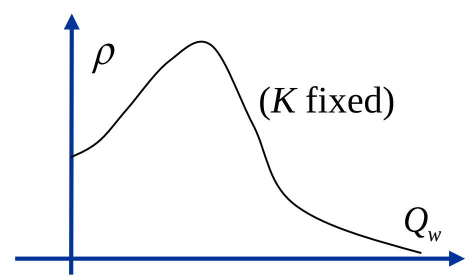
As $K \to \infty$, $\rho = Ge^{-G}$ *as before.*





waiting time (not including successful transmissions) $= \boxed{W = S\left(\frac{1-q_a}{q_r}\right)\left(r + \frac{K+1}{2}\right)}$

Av. waiting queue length, $\boxed{Q_w = \frac{\rho}{S} \cdot W = \rho \cdot \left(\frac{1-q_a}{q_r}\right)\left(r + \frac{K+1}{2}\right)}$



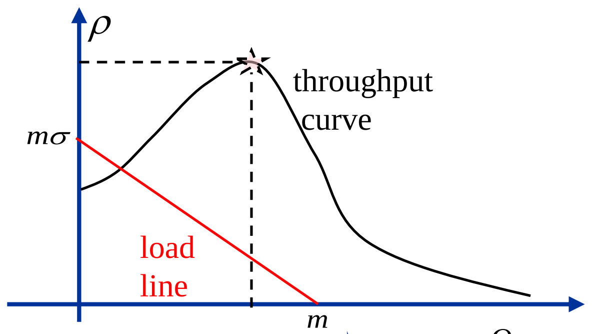
Stability Issues - 1

Suppose have m users

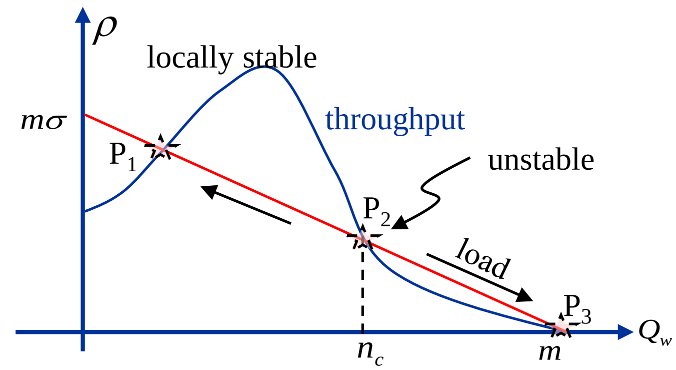
Let $\sigma = pr\{\text{a node generates a request}\}$

If Q_w are backlogged, then

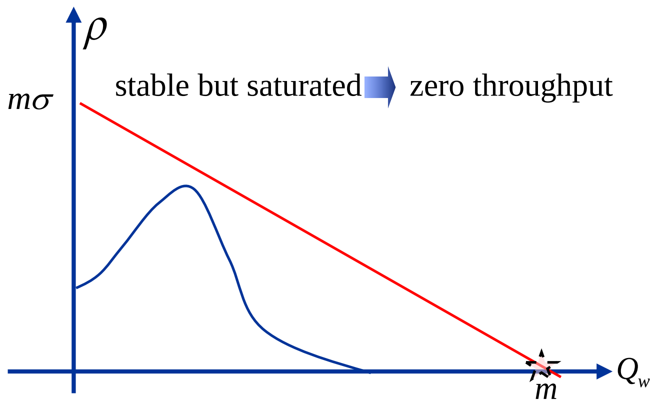
$\rho = (m\text{-}Q_w)\sigma$ = input packet rate per packet length

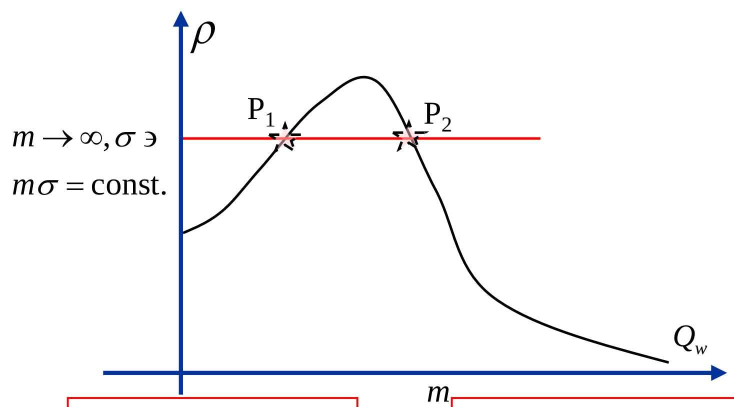


throughput > load ⇒ stable



Bistable behavior “unstable at P_3 since the channel is saturated”



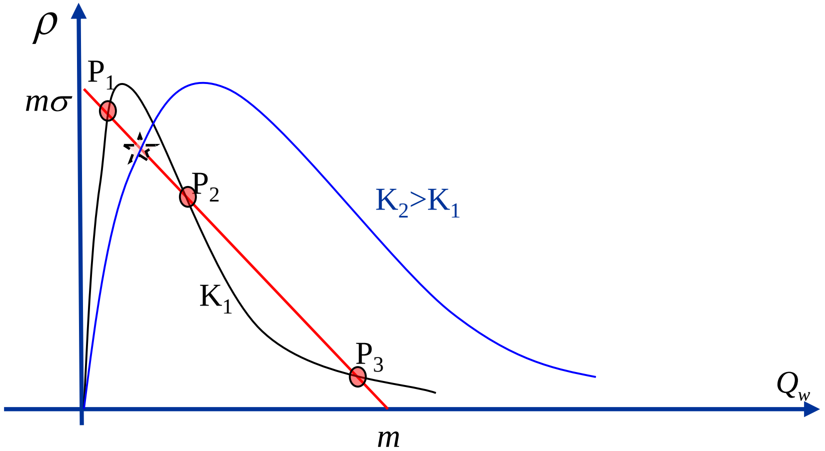


P_1=locally stable
P_2=locally unstable

⇒ Infinite population slotted Aloha is unstable

Stability Issues - 2

- We can make it stable by increasing *K*.



- **Stabilization of slotted Aloha:**

Suppose have m users and each attemps to transmit a packet with prob. σ.

If Q_w *is the backlog, then new attempt rate* $= (m - Q_w)\sigma$

The backlogged packets attempt at a rate $\gamma \Rightarrow \Pr ob(retransmission) = \gamma$. *Then*

$G(Q_w) = (m - Q_w)\sigma + Q_w\gamma = \rho + Q_w\gamma$ *as* $m \to \infty$

We can control γ *to stabilize the system. But, don't know* Q_w.

$\Rightarrow$ *Estimate* Q_w *online based on success and failure rates of packets.*

Stabilization of Slotted Aloha

1) Suppose want to keep $G \approx 1$, then

$$know \;\; \rho \approx Ge^{-G} \quad \text{and } \rho_{max} = \frac{1}{e} \;\; \text{at } G = 1$$

$$prob\{idle\ \text{slot}\} = e^{-G} = e^{-1} \cong .368$$

$$prob\{successful\ transmission\} = \rho = e^{-1} \cong .368$$

$$prob\{collision\} = 1 - \frac{2}{e} \cong .264$$

So, if you know *n* exactly, can control *G(n)*, $\ni G(n) \approx 1$

and achieve $\rho \approx \frac{1}{e} \;\; \Rightarrow \text{need an estimator of } n$

The above assume that all nodes use the same retransmission rate, γ

2) We can get better throughput (i.e., $\rho_{max} > \frac{1}{e}$) if each node keeps track of its own history of retransmissions and the feedback history (idle, success, collision) → Splitting Algorithms

Pseudo-Bayesian Algorithm -1

- **"Pseudo Bayesian" Algorithm**

Assumption: new and collided packets are assumed to be backlogged

If there are *n* packets (including new arrivals) at the beginning of a slot, the attempt rate $= n\gamma$

$$\textit{success}\ \text{prob.} = n\gamma(1-\gamma)^{n-1}$$

But, don't know *n* and needs to be estimated online based on the knowledge that
1) The previous slot is idle or a packet was successfully transmitted
2) There was a collision in the previous slot.

If have $\hat{n}$, we set $\boxed{\gamma = \min\{1, \frac{1}{\hat{n}}\} \text{ so that } G(n) \cong 1}$

- How to get the estimate $\hat{n}$?

Suppose that the prior probability of the number of backlogged packets *n* at slot *k* is Poisson with mean $\hat{n}_k$ (i.e., just before we know what happened in slot *k*)

$$p(n) = \frac{(\hat{n}_k)^n e^{-\hat{n}_k}}{n!}$$

Pseudo-Bayesian Algorithm -2

- Suppose slot k is idle, then

$$p(n \mid slot\ k\ is\ idle) = \frac{p(slot\ k\ is\ idle \mid n)p(n)}{p(slot\ k\ is\ idle)}$$

Since each node transmits with probability $\dfrac{1}{\hat{n}_k}$

$$prob\{slot\ k\ idle \mid n\} = (1-\frac{1}{\hat{n}_k})^n$$

$$prob\{slot\ k\ idle\} = \sum_{n=0}^{\infty}(1-\frac{1}{\hat{n}_k})^n \frac{e^{-\hat{n}_k}}{n!}(\hat{n}_k)^n = e^{-1}$$

So,

$$p(n \mid slot\ k\ is\ idle) = \frac{(1-\frac{1}{\hat{n}_k})^n e^{-\hat{n}_k}(\hat{n}_k)^n}{n!e^{-1}} = \frac{e^{-(\hat{n}_k-1)}(\hat{n}_k-1)^n}{n!}$$

$$= Poisson\ with\ mean\ \hat{n}_k\text{-}1$$

$$E(n \mid slot\ k\ is\ idle) = \hat{n}_k\text{-}1$$

Pseudo-Bayesian Algorithm - 3

- Similarly,

$$p(n+1|\,slot\ k\ is\ successful)\ =\ poisson\ with\ \ mean\ \hat{n}_k\text{-}1$$

$$\therefore\ E(n|\ slot\ k\ is\ successful)\ =\ \hat{n}_k\text{-}1$$

$$p(n|collision\ in\ slot\ k)=\frac{p(collision\ in\ slot\ k|n)p(n)}{p(collision\ in\ slot\ k)}$$

$$=\ \frac{e}{e\text{-}2}[\frac{e^{-\hat{n}_k}(\hat{n}_k)^n}{n!}-\frac{e^{-\hat{n}_k}(\hat{n}_k-1)^n}{n!}-\frac{e^{-\hat{n}_k}(\hat{n}_k-1)^{n-1}}{(n-1)!}]$$

Not poisson, but assume it anyway

$$\therefore\ \mathrm{E(n\,|\,collision\,in\,slot\ k)}=\ \frac{e}{e\text{-}2}\hat{n}_k-\frac{1}{e-2}(\hat{n}_k-1)-\frac{1}{e-2}\hat{n}_k$$

$$=\ \hat{n}_k+\frac{1}{e\text{-}2}$$

Pseudo-Bayesian Algorithm - 4

So, $$\hat{n}_{k+1} = \begin{cases} \max\{\rho, \hat{n}_k + \rho - 1\} & \textit{if idle or success} \\ \hat{n}_k + \rho + \dfrac{1}{e-2} & \textit{if collision} \end{cases}$$

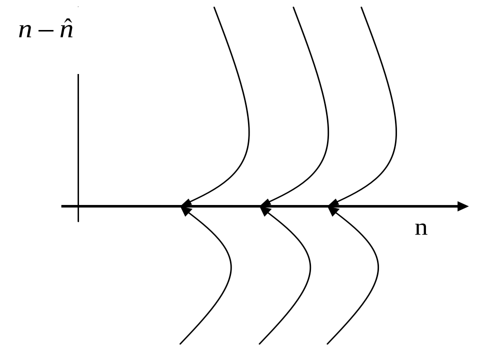
where ρ accounts for new arrival during slot k

Let us look at the stability of the system for $\lambda < \dfrac{1}{e}$

State of the system is $(n, \hat{n})$ *or* $(\mathrm{n}, \mathrm{n} - \hat{\mathrm{n}})$

$Case\,1: n \approx \hat{n}$ *and large*

$$E(n_{k+1} - n_k) = \rho - \frac{1}{e} < 0$$



$$E(\hat{n}_{k+1} - \hat{n}_k) = \frac{2}{e}(\rho - 1) + (1 - \frac{2}{e})(\rho + \frac{1}{e-2})$$

$$= \frac{2}{e}(\rho - 1) + \frac{1}{e} + (e-2)\frac{\rho}{e} = \rho - \frac{1}{e} < 0 \Rightarrow av.\, drift < 0$$

Pseudo-Bayesian Algorithm - 5

- Case 2: If $|n-\hat{n}|$ is large $\Rightarrow \gamma$ is too high or too low depending on whether $(n-\hat{n})>0$ or $(n-\hat{n})<0$

$(n-\hat{n}) >> 0 \Rightarrow$ You may decide to send at a prob more than necessary

$$\Rightarrow \textit{more collisions} \Rightarrow n\uparrow \textit{but } (n-\hat{n})\downarrow \textit{faster}$$

$(n-\hat{n}) << 0 \Rightarrow$ You may decide not to send

$$\Rightarrow \textit{idle slots} \Rightarrow n\uparrow \textit{ due to arrivals, but } E|n\text{-}\hat{n}|\downarrow \textit{faster}$$

$$\textit{Eventually } n \to \hat{n} \textit{ and } n\downarrow$$

- Binary Exponential Backoff
  - feedback on own packets only
  - set retransmission probability, $\gamma = 2^{-i}$, i = number of failures
  - used extensively in ethernet

Splitting Algorithms

- Splitting algorithms:

All of them have some form of *tree structure* to resolve conflicts.
Suppose a collision occurs in slot k, then the collisions are resolved as follows:

- All nodes not involved in the collision go into a waiting mode
- All nodes involved in the collision do the following
  - Split into two subsets (e.g., by flipping a coin)

Note: this splitting may also be based on time of arrival

The first subset transmits in slot (k+1)

If slot is idle or successful

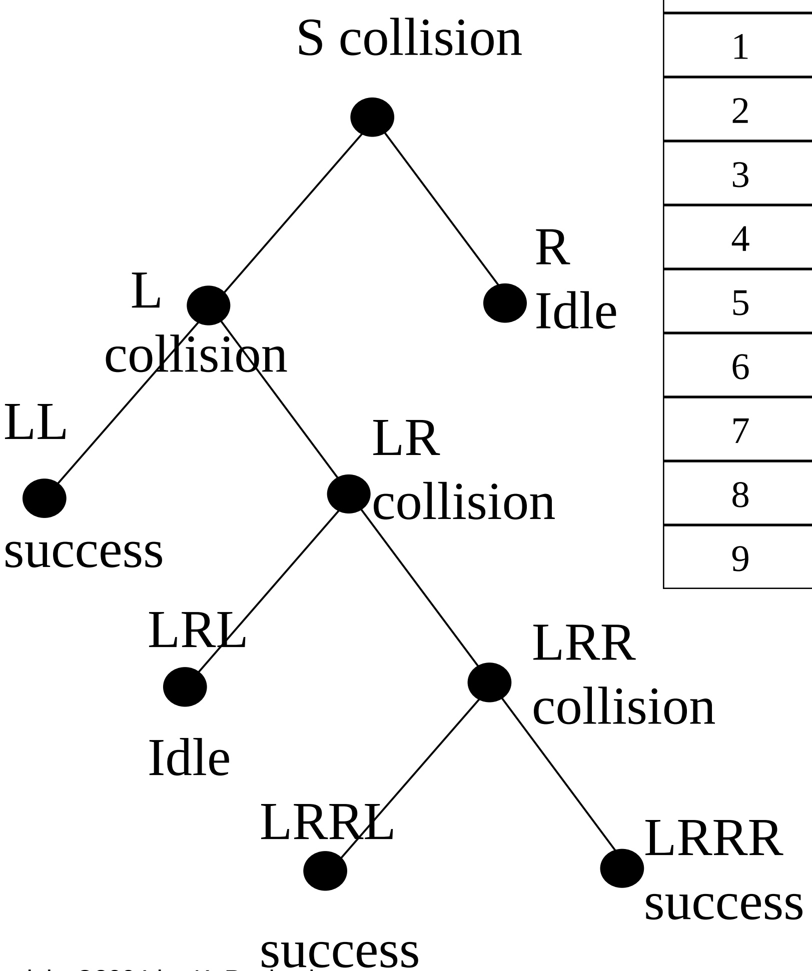
second subset transmits in slot (k+2)

else (i.e., collision)

split again and continue

Tree Algorithms -1

3 packets



slot	transmit set	Waiting set	Feedback
1	S	---	E (error)
2	L	R	E
3	LL	LR, R	1 (success)
4	LR	R	E
5	LRL	LRR, R	0 (idle)
6	LRR	R	E
7	LRRL	LRRR, R	1
8	LRRR	R	1
9	R	---	0

Tree Algorithms - 2

- We can implement this algorithm using a stack.
- A node can keep track of when to transmit

$$\textit{packet}\text{ collided} \Rightarrow \text{set counter to } \begin{matrix} 0 \text{ or } 1 \\ \text{L} \quad \text{R} \end{matrix}$$

$$\textit{counter} = 0 \qquad \Rightarrow \text{transmit}$$

$$\text{counter} \neq 0 \qquad \Rightarrow \text{counter} = \begin{cases} \text{counter}+1; & \text{for collision} \\ \text{counter}-1; & \text{for idle or success} \end{cases}$$

- What to do with a new packet?
  - Wait until the <u>collision resolution period</u> (CRP) ends

CRP is large $\Rightarrow$ lot of arrivals $\Rightarrow$ larger CRP and so on.

<u>Solution</u>: split the nodes with arrivals into j subsets, where j is chosen such that E{# of elements in a subset} is slightly greater than 1.

Place subsets in a stack and start the new CRP.

Capetanakis: Max throughput = 0.43 packets/slot

Improved Tree Algorithms - 1

- Improvements to the tree algorithm:

1) Collision followed by an idle slot $\Rightarrow$ one subset is null and the other subset is the complete subset. So, collision LRR is avoidable. To improve throughput,

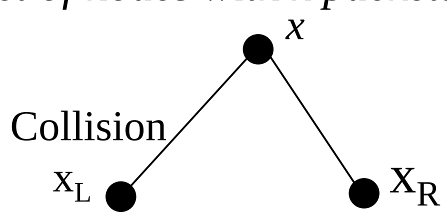
- *omit transmission of second subset*
- *split it into two subsets*
- *transmit the first of the split subsets*
- *If an idle occurs, split the second subset again*

This can be easily accomplished by each node by having an extra bit to keep track of idle slots following collisions.

This improves maximum throughput to 0.46.

2) *Suppose have a collision followed by a collision*

subset of nodes with x packets, collision $\Rightarrow x_L + x_R \geq 2$



$$x_L + x_R = x$$

Improved Tree Algorithms - 2

- x_L and x_R are Poisson if x is Poisson : recall splitting property

Collision implies $x_L \geq 2$

So,

$$P(x_R = i \mid x_L \geq 2, x_L + x_R \geq 2) = P(x_R = i \mid x_L \geq 2)$$

Since expected number of packets for x_R is "small", treat x_R as if they are new arrivals $\Rightarrow$ they are not part of current CRP

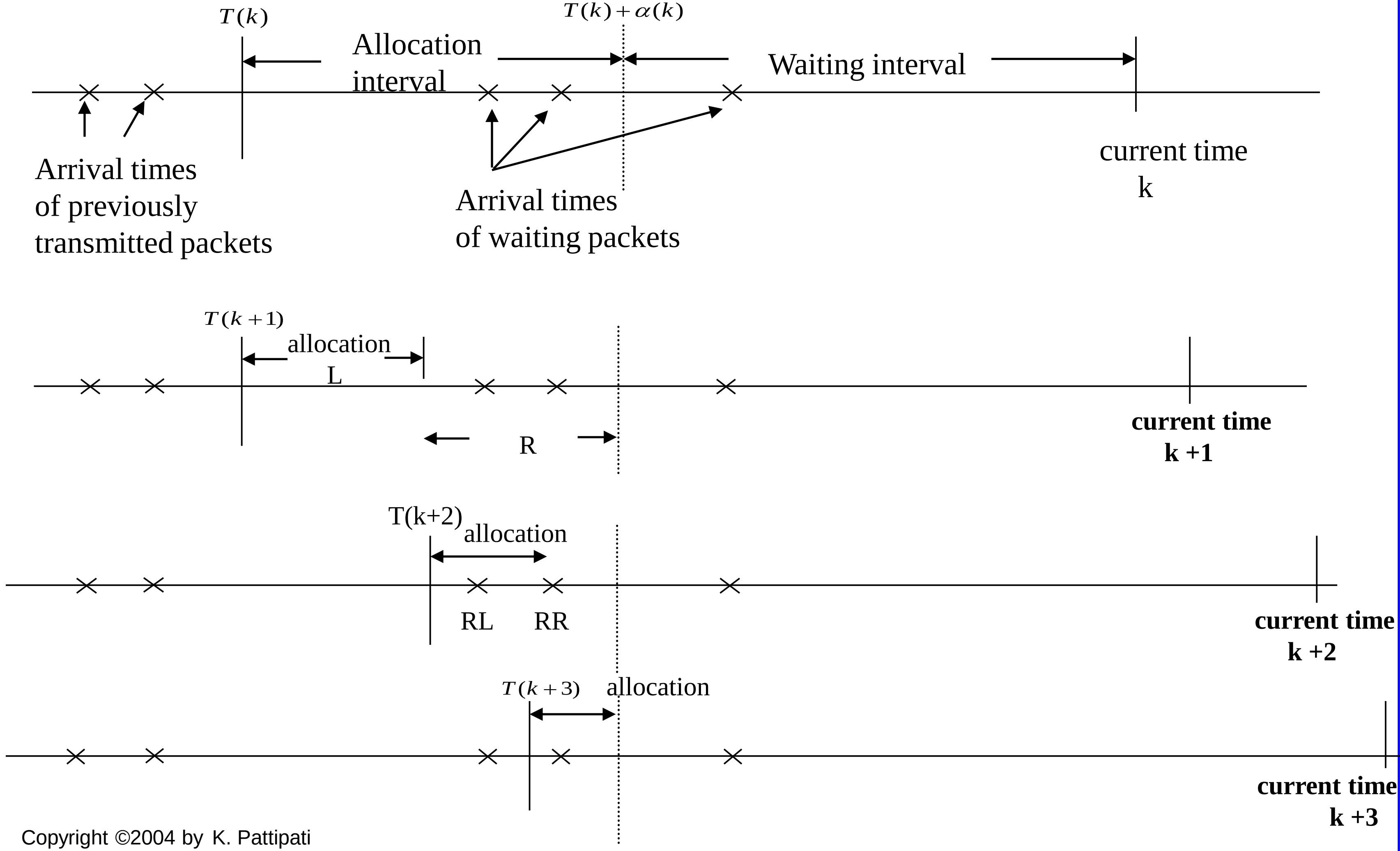
- FCFS splitting algorithm

"Split the subset on the basis of arrival intervals"

At each time slot k, the algorithm specifies the packets to be transmitted to be the set of packets that arrived in some earlier interval $(T(k),\ T(k) + \alpha(k))$

FCFS Splitting Algorithm - 1

$(T(k), \mathrm{T(k)} + \alpha(\mathrm{k}))$ is the allocation interval



FCFS Splitting Algorithm - 2

- *packets* arriving after $T(k)+\alpha(k)$ are in the queue (waiting).
- *packets arriving during* $[T(k), T(k)+\alpha(k)]$ *are in service. But, don't know the # of packets.*

If collision, split allocation interval into two parts and assign left-most subinterval as the allocation subinterval to slot (k+1)

$$T(k+1) = T(k)$$
$$\alpha(k+1) = \frac{\alpha(k)}{2}$$
transmit packets in left subinterval

If success and was transmitting left subinterval packets

$$T(k+1) = T(k) + \alpha(k)$$
$$\alpha(k+1) = \alpha(k)$$
transmit packets in right subinterval

FCFS Splitting Algorithm - 3

If idle and was transmitting left subinterval $\Rightarrow$ split right - most interval

$$T(k+1) = T(k) + \alpha(k)$$

$$\alpha(k+1) = \frac{\alpha(k)}{2}$$

Transmit left subinterval packets (i.e., left subinterval of split right subinterval)

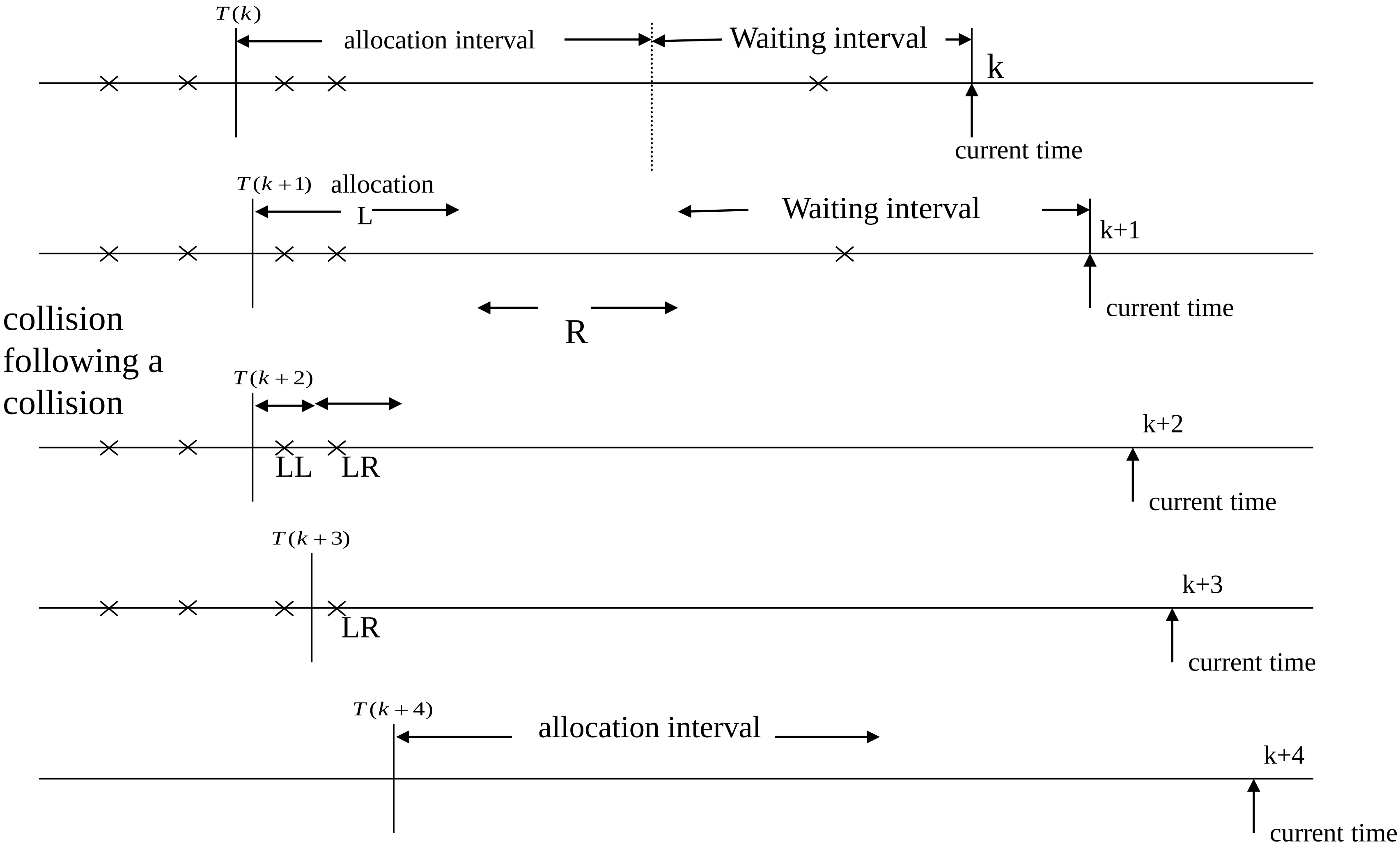
If idle or success and was transmitting right

$$T(k+1) = T(k) + \alpha(k)$$

$$\alpha(k+1) = \min(\alpha_0, k+1-T(k+1))$$

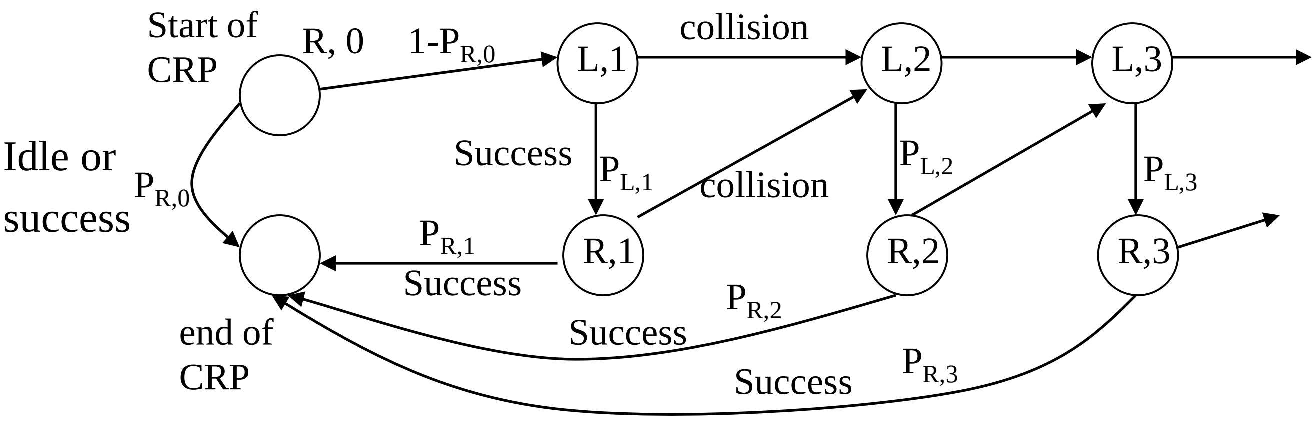
Transmit packets in right subinterval

FCFS Splitting Algorithm - 4



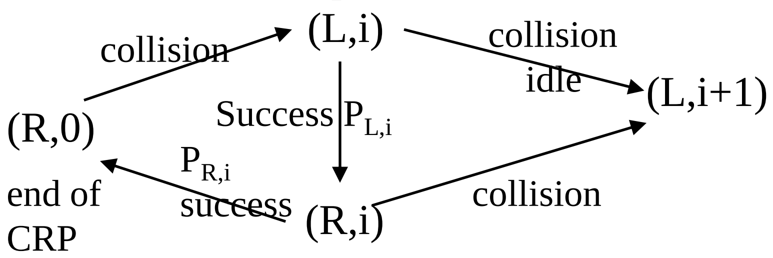
Markov Chain Analysis -1

- Markov Chain Representation of Splitting Algorithm:



If a collision occurs (R,0) → (L,1)

left, one split



Markov Chain Analysis - 2

- Each split decreases the allocation interval by a factor of 2.

  i splits $\Rightarrow \alpha_0 \rightarrow 2^{-i}\alpha_0$

- Average # of packets in the allocation interval $L_i = 2^{-i}\rho\alpha_0$

$$P_{R0} = prob\{\,idle \text{ or success}\}$$
$$= (1 + \mathrm{L}_0)\mathrm{e}^{-\mathrm{L}_0}$$

- Transition from (L,1) → (R,1) occurs if packet is successfully transmitted

$$P_{L,1} = \frac{prob\{x_L = 1\}\,prob\{x_R \geq 1\}}{prob\{x_L + x_R \geq 2\}} = \frac{L_1 e^{-L_1}(1 - e^{-L_1})}{[1 - (1 + L_0)e^{-L_0}]}$$

$$similarly,\;\; P_{R,1} = \frac{P(x_R = 1)}{P(x_R \geq 1)} = \frac{L_1 e^{-L_1}}{1 - e^{-L_1}}$$

Markov Chain Analysis - 3

- In general,

$$P_{L,i} = \frac{L_i e^{-L_i}(1-e^{-L_i})}{[1-(1+L_{i-1})e^{-L_{i-1}}]}$$

$$P_{R,i} = \frac{L_i e^{-L_i}}{1-e^{-L_i}}$$

- Prob of Markov states: $P(L,1) = 1 - P_{R,0}$

$$P(R,i) = P_{L,i}P(L,i),\ i \geq 1$$

$$P(L,i+1) = (1 - P_{L,i})P(L,i) + (1 - P_{R,i})P(R,i)$$

$$= P(L,i) - P_{L,i}P(L,i) + P_{L,i}P(L,i) - P_{R,i}P_{L,i}P(L,i)$$

$$\Rightarrow P(L,i+1) = (1 - P_{R,i}P_{L,i})P(L,i),\quad P(L,0) = 1, P_{L,0} = 1$$

If we let $E\{K\}$ be the average number of slots in a CRP, then

$$E\{K\} = 1 + \sum_{i=1}^{\infty}[P(L,i) + P(R,i)]$$

Markov Chain Analysis - 4

Note: $P(L,i+1)+P(R,i+1)=[1+P_{L,i}(1-P_{R,i})]P(L,i)$

Recall that

$$1+P_{L,i}(1-P_{R,i})=\frac{1-(1+L_{i-1})e^{-L_{i-1}}+L_i e^{-L_i}[1-(1+L_i)e^{-L_i}]}{1-(1+L_{i-1})e^{-L_{i-1}}}$$

$$=1+L_i e^{-L_i}P\{collision\,|\,state(L,i)\}$$

- Change in *T(k)* from one CRP to the next

  Initial allocation interval α_0

  If left hand intervals have collisions, then the corresponding right hand intervals are returned to the waiting interval.

  If f is the fraction that is returned to waiting interval then change in $T(k)=\alpha_0(1-f)$

$$Prob\ of\ collision\ in\ state\ (L,i)$$

$$=prob\{left\ hand\ interval\ has\ at\ least\ two\ packets\ |\ right+left\geq 2\}$$

$$=\frac{1-(1+L_i)e^{-L_i}}{1-(1+L_{i-1})e^{-L_{i-1}}}=P(e\,|(L,i))$$

Markov Chain Analysis - 5

The fraction of the original interval returned on such a collision is 2^{-i}

So
$$E(f) = \sum_{i=1}^{\infty} P(L,i) P(e \mid (L,i)) 2^{-i}$$

So, $\quad E\{K\}$ *and* $E\{f\}$ *are functions of* L_i *or of* $\rho\alpha_0$

$$\textit{Drift},\ D = E\{K\text{-}T(k)\}\ \textit{over a CRP}$$
$$D = E\{K\}\text{-}\alpha_0[1 - E(f)]$$

$$D < 0\ (\Rightarrow \textit{stable})\quad \textit{if}\ \rho < \frac{\rho\alpha_0[1 - E\{f\}]}{E\{K\}}$$

$$Max\ \rho = 0.4871 \text{ at } \rho\alpha_0 = 1.266 \ \text{(Numerical evaluation)}$$
$$\text{pick } \alpha_0 = 2.6$$

Summary

- Random Access Networks
- Pure and Slotted Aloha
- Stability Issues
- Stabilization of Slotted Aloha
- Splitting (Tree) Algorithms
- Markov Chain Analysis
- Summary