The University of Connecticut **Fall 2009**
Dept. of ECE **KRP**

<u>Problem Set # 2</u>
(Due Date: September 15, 2009)

1. A company has n factories. Factory i is located at the point (x_i, y_i) in the x-y plane. The company wants to locate a warehouse at a point (x, y) that minimizes

$$\min_{x,y} \sum_{i=1}^{n} \left[|x - x_i| + |y - y_i| \right]$$

   Find the optimal location of the warehouse. Interpret the solution.

2. Find all local maxima, local minima, and saddle points for $f(x_1, x_2) = x_1^3 - 3x_1x_2^2 + x_2^4$.
3. In a Basketball league, the following games have been played: team A beats team B by 7 points, team C beats team A by 8 points, team B beats team C by 6 points, and team B beats team C by 9 points. Let r_A, r_B, r_C represent "ratings" for each team in the sense that if, say, team A plays team B, we predict that team A will defeat team B by ($r_A - r_B$) points. Determine the values of r_A, r_B, and r_C that best fit (in the least-squares sense) these results. To obtain a unique set of ratings, it may be helpful to add the constraint ($r_A + r_B + r_C$) = 0. This ensures that an "average" team will have a rating of 0.
4. Consider a second order Taylor series expansion of $f(\underline{x}_k + \alpha \underline{d}_k)$ around $\underline{x}_k$, where $\underline{d}_k$ is a descent direction. Assuming that the Hessian $\left[\partial^2 f / \partial x_i \partial x_j\right] > 0$, find the optimum α^*. What is $f(\underline{x}_k + \alpha^* \underline{d}_k)$? Specialize the results to the following three cases: (a) Steepest descent, (b) Newton's method, and (c) General gradient method, $\underline{d}_k = -H_k \underline{g}_k$, where $\underline{g}_k$ is the gradient of $f(\underline{x})$ at $\underline{x} = \underline{x}_k$.
5. Consider a second order Taylor series expansion of $f(\underline{x}_k + \underline{d}_k)$ around $\underline{x}_k$. If you could choose $\underline{d}_k$, what would you pick to achieve maximal decrease in f. Assume that $\left[\partial^2 f / \partial x_i \partial x_j\right] > 0$.
6. Let $f(\underline{x}) = 3x_1^2 + 2x_1x_2 + x_2^2$. Let the initial point be $\underline{x}_0 = (1,\ 1)^T$. Find $\underline{x}_1$ for the following methods: (a) Newton, (b) Diagonal Newton, $H_k = \text{Diag}\left(1/\partial^2 f / \partial x_i^2\right)$, and (c) $H_k = \left[F(\underline{x}_k) + \mu I\right]^{-1}$, where $F(\underline{x}_k)$ is the Hessian and $\mu = 10$.
7. Let $f(\underline{x}) = x^2$. Show that the infinite sequence of points $x_i = (-1)^i \left[1 + 2^{-i}\right], i = 0,\ 1,\ 2\ \ldots$ is not permitted by the Armijo step size rule for any $\sigma > 0$.
8. Apply Fibonacci search to the scalar function $f(x) = x^2 - 6x + 2$ given that $l_1 = 0$, and $r_1 = 10$. It is desired that $(r_N - l_N) \le 0.1(r_1 - l_1)$.