The University of Connecticut **Fall 2009**
Dept. of ECE **KRP**

<u>Problem Set # 4</u>

(Due September 29, 2009)

1. *(What is the convergence rate of Modified Newton method?)*

   Find an estimate for the rate of convergence for the modified Newton method

   $$\underline{x}_{k+1} = \underline{x}_k - \alpha_k (\varepsilon_k I + F_k)^{-1} \underline{g}_k$$

   where ε_k is chosen such that the matrix $(\varepsilon_k I + F_k)$ has eigenvalues greater than or equal to δ. Assume that the step size α_k is optimal.

2. Let H be an nxn symmetric positive definite matrix, and let $\{\underline{v}_i\}_{i=1}^n$ be the given eigenvectors with the corresponding eigenvalues $\{\lambda_i\}_{i=1}^n$. Show that

   $$If\ \underline{g} = \sum_{i=1}^{n} \alpha_i \underline{v}_i\ then\ (H + \mu I)^{-1} \underline{g} = \sum_{i=1}^{n} (\frac{\alpha_i}{\lambda_i + \mu}) \underline{v}_i$$

3. Let $f(\underline{x}) = \frac{1}{2} x_1^2 + x_2^2, \underline{x}_0 = (1,1)^T$.

   a) What is the value of μ to satisfy $\| (F_0 + \mu I)^{-1} \underline{g}_0 \| = \delta$ for $\delta = 2$ and $\delta = 5/6$, where F_0 and $\underline{g}_0$ are the Hessian and the gradient at $\underline{x}_0$, respectively.

   b) Use double dogleg step to compute $\underline{x}_1$ for $\delta = 1,\ 5/4$ and 2.

4. Show that the Gauss-Newton method minimizes the quadratic cost function:

   $$\frac{1}{2} \left\| \underline{g}(\underline{x}_k) + J^T(\underline{x}_k)(\underline{x}^* - \underline{x}_k) \right\|^2$$