The University of Connecticut **Fall 2009**
Dept. of ECE **KRP**

Problem Set # 9
(Due Date: November 17, 2009)

1. Use the QPP method to solve the three dimensional quadratic problem

$$\min\ f(\underline{x}) = x_1^2 + 2x_2^2 + 3x_3^2$$
$$\text{subject to } x_1 + x_2 + x_3 \geq 1, \quad x_i \geq 0, i = 1,2,3$$

Starting from the point $\underline{x}^T(0) = [0 \quad 0 \quad 1]$.

2. Use the Lagrange multiplier theorem to solve the following *minimization* problems:

a. $f(\underline{x}) = \|\underline{x}\|_2^2,\ h(\underline{x}) = \sum_{i=1}^{n} x_i - 1 = 0$

b. $f(\underline{x}) = \sum_{i=1}^{n} x_i,\ h(\underline{x}) = \|\underline{x}\|_2^2 - 1 = 0$

c. $f(\underline{x}) = \|\underline{x}\|_2^2,\ h(\underline{x}) = \underline{x}^T Q \underline{x} - 1 = 0, \text{where } Q \text{ is PD}$

d. $f(\underline{x}) = \sum_{i=1}^{n} x_i,\ h(\underline{x}) = \prod_{i=1}^{n} x_i = 1, x_i > 0, i = 1,2,..,n$

Use this to show

$$\left(\prod_{i=1}^{n} x_i\right)^{1/n} \leq \frac{\sum_{i=1}^{n} x_i}{n}$$ **for a set of positive numbers** $x_i, i = 1,2,..,n$

3. Redo of Problem 5(b) of Problem set # 6 in a more general way. Given a vector $\underline{y}$, consider the problem $\text{maximize } \underline{y}^T \underline{x} \text{ subject to } \underline{x}^T Q \underline{x} \leq 1$ where Q is PD. Show that the optimal value is $\sqrt{\underline{y}^T Q^{-1} \underline{y}}$ and use this fact to establish the inequality $\left(\underline{x}^T \underline{y}\right)^2 \leq \left(\underline{x}^T Q \underline{x}\right)\left(\underline{y}^T Q^{-1} \underline{y}\right)$.

4. Consider the following problem:

$$\min -2x_1 + x_2$$
$$\text{s.t.} \quad x_1 + x_2 - 3 = 0 \ \& \left(x_1, x_2\right) \in \Omega$$
$$\Omega = \{(0,0),(0,4),(4,4),(4,0),(1,2),(2,1)\}$$

a) Solve the primal problem?
b) Write the dual and find the optimal dual objective function?
c) Explain why (a) and (b) are not the same.