HW set # 6 Solution

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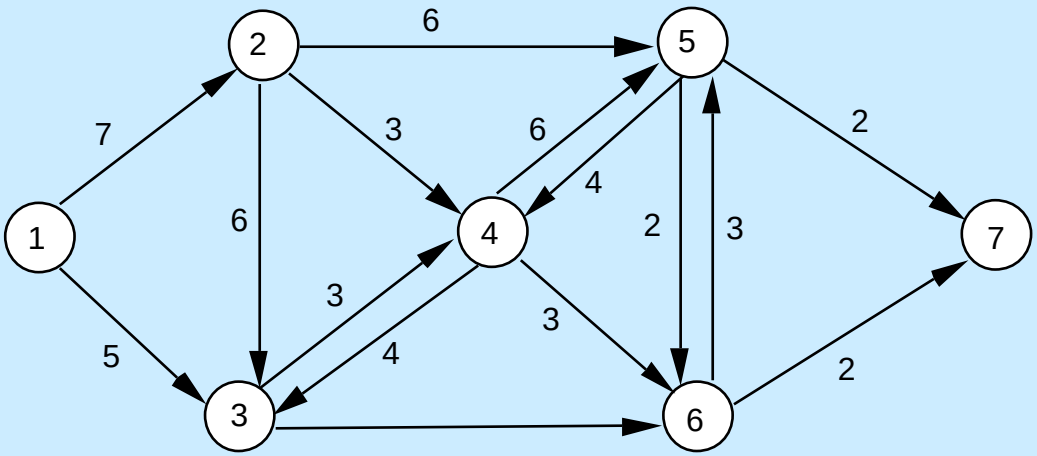
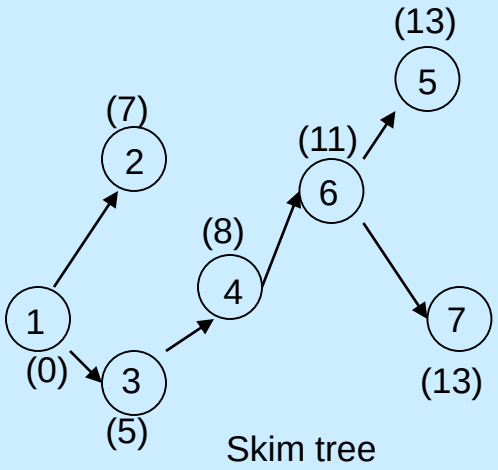
ECE 6108
Linear Programming and Network Flows

Problem 1





Dijkstra: Initialize:

$\lambda_1 = 0; \lambda_2 = 7; \lambda_3 = 5; \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \infty$

$\Rightarrow \underline{\lambda} = [0,7,5,\infty,\infty,\infty,\infty]; W = \{1\}; T = \{2,3,4,5,6,7\}$

$Iteration1: W = \{1,3\}; T = \{2,4,5,6,7\}$

$\lambda_2 = \min(7,5+\infty) = 7; \lambda_4 = \min(\infty,5+3) = 8; \lambda_5 = \min(\infty,5+\infty) = \infty;$

$\lambda_6 = \min(\infty,5+9) = 14; \lambda_7 = \min(\infty,5+\infty) = \infty$

$\Rightarrow \underline{\lambda} = [0,7,5,8,\infty,14,\infty]$

$Iteration2: W = \{1,3,2\}; T = \{4,5,6,7\}$

$\lambda_4 = \min(8,7+3) = 8; \lambda_5 = \min(\infty,7+10) = 17;$

$\lambda_6 = \min(14,7+\infty) = 14; \lambda_7 = \min(\infty,7+\infty) = \infty$

$\Rightarrow \underline{\lambda} = [0,7,5,8,17,14,\infty]$

$Iteration3: W = \{1,3,2,4\}; T = \{5,6,7\}$

$\lambda_5 = \min(17,8+6) = 14; \lambda_6 = \min(14,8+3) = 11;$

$\lambda_7 = \min(\infty,8+\infty) = \infty$

$\Rightarrow \underline{\lambda} = [0,7,5,8,14,11,\infty]$

$Iteration4: W = \{1,3,2,4,6\}; T = \{5,7\}$

$\lambda_5 = \min(14,11+2) = 13; \lambda_7 = \min(\infty,11+2) = 13;$

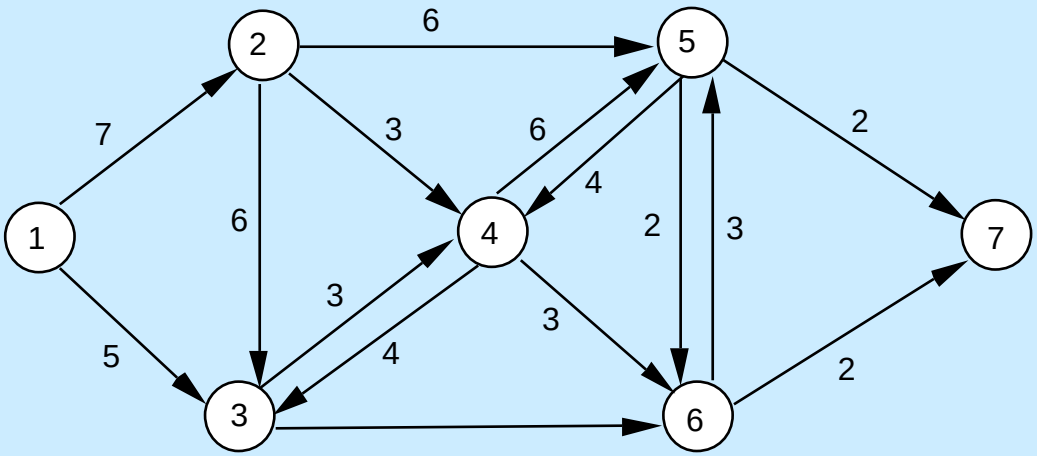
$\Rightarrow \underline{\lambda} = [0,7,5,8,13,11,13]$

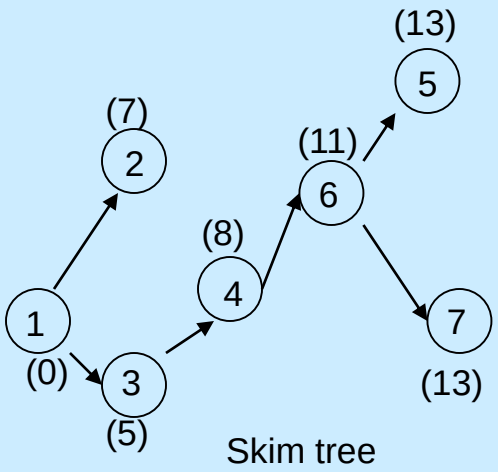
$Iteration5: W = \{1,3,2,4,6,5\}; T = \{7\}$

$\lambda_7 = \min(13,13+2) = 13$

$\Rightarrow \underline{\lambda} = [0,7,5,8,13,11,13]$

- Problem 1





BMDP: Initialize:

$\lambda_1 = 0; \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \infty$

$\Rightarrow \underline{\lambda} = [0, \infty, \infty, \infty, \infty, \infty, \infty]; Q = \{1\}$

$Iteration1: \Pr ed(2) = 1; \Pr ed(3) = 5; \underline{\lambda} = [0,7,5, \infty, \infty, \infty, \infty], Q = \{2,3\}$

$Iteration2: \Pr ed(5) = 2; \Pr ed(4) = 2; \underline{\lambda} = [0,7,5,10,17, \infty, \infty], Q = \{3,4,5\}$

$Iteration3: \Pr ed(4) = 3; \Pr ed(6) = 3; \underline{\lambda} = [0,7,5,8,17,14, \infty], Q = \{4,5,6\}$

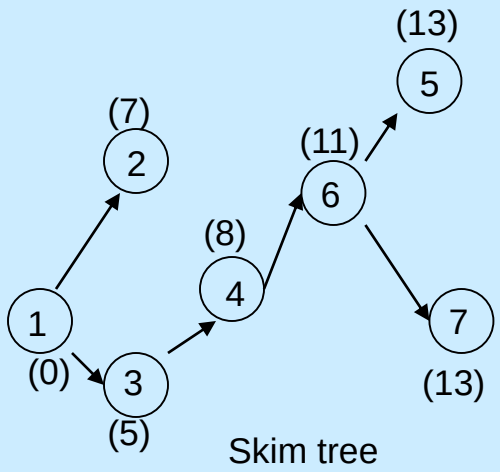
$Iteration4: \Pr ed(5) = 4; \Pr ed(6) = 4; \underline{\lambda} = [0,7,5,8,14,11, \infty], Q = \{5,6\}$

$Iteration5: \Pr ed(7) = 5; \underline{\lambda} = [0,7,5,8,14,11,16], Q = \{6,7\}$

$Iteration6: \Pr ed(5) = 6; \Pr ed(7) = 6; \underline{\lambda} = [0,7,5,8,13,11,13], Q = \{7\}$

$Iteration7:$ Same as iteration 6. $\underline{\lambda} = [0,7,5,8,13,11,13], Q = \{\phi\}$

- Problem 2



Skim tree

BMDP: Initialize:

$\lambda_1 = 0; \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \infty$

$\Rightarrow \underline{\lambda} = [0, \infty, \infty, \infty, \infty, \infty, \infty]; Q = \{1\}$

$Iteration1: \Pr ed(2) = 1; \Pr ed(3) = 5; \underline{\lambda} = [0,7,5, \infty, \infty, \infty, \infty], Q = \{2,3\}$

$Iteration2: \Pr ed(5) = 2; \Pr ed(4) = 2; \underline{\lambda} = [0,7,5,10,17, \infty, \infty], Q = \{3,4,5\}$

$Iteration3: \Pr ed(4) = 3; \Pr ed(6) = 3; \underline{\lambda} = [0,7,5,8,17,14, \infty], Q = \{4,5,6\}$

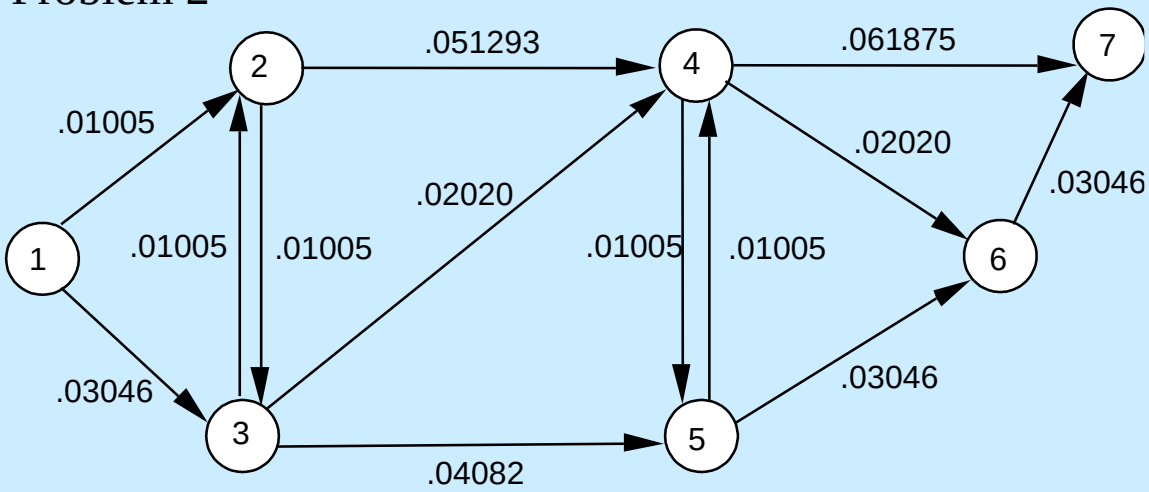
$Iteration4: \Pr ed(5) = 4; \Pr ed(6) = 4; \underline{\lambda} = [0,7,5,8,14,11, \infty], Q = \{5,6\}$

$Iteration5: \Pr ed(7) = 5; \underline{\lambda} = [0,7,5,8,14,11,16], Q = \{6,7\}$

$Iteration6: \Pr ed(5) = 6; \Pr ed(7) = 6; \underline{\lambda} = [0,7,5,8,13,11,13], Q = \{7\}$

$Iteration7:$ Same as iteration 6. $\underline{\lambda} = [0,7,5,8,13,11,13], Q = \{\phi\}$

- Problem 2



$$c_{ij} = -\ln(1 - p_{ij})$$

Dijkstra: Initialize:

$\lambda_1 = 0; \lambda_2 = .01005; \lambda_3 = .03046; \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \infty$

$\Rightarrow W = \{1\}, T = \{2,3,4,5,6,7\}$

$Iteration1: W = \{1,2\}, T = \{3,4,5,6,7\}$

$\lambda_3 = \min(.03046, .0201) = .0201; \lambda_4 = .061343; \lambda_5 = \lambda_6 = \lambda_7 = \infty$

$Iteration2: W = \{1,2,3\}, T = \{4,5,6,7\}$

$\lambda_4 = \min(.061343, .02010 + .02020) = .0403; \lambda_5 = .06092; \lambda_6 = \lambda_7 = \infty$

$Iteration3: W = \{1,2,3,4\}, T = \{5,6,7\}$

$\lambda_5 = .05035; \lambda_6 = .0605; \lambda_7 = .102175$

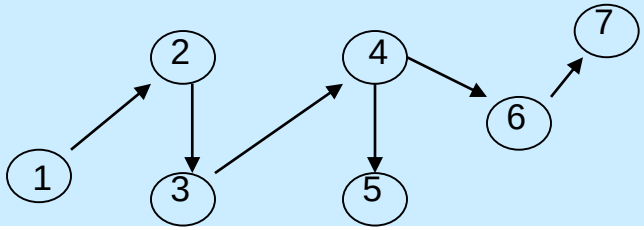
$Iteration4: W = \{1,2,3,4,5\}, T = \{6,7\}$

$\lambda_6 = .0605; \lambda_7 = .102175$

$Iteration5: W = \{1,2,3,4,5,6\}, T = \{7\}; \lambda_7 = .09096$

$$P\{success\} = \prod_{(ij) \in shortest\ path} (1 - p_{ij}) = 0.90904 = 0.909$$

Skim tree



- Problem 3

$Step1: Initialize\ \lambda_1 = 0; \lambda_j = c_{ij} \forall j = 2,3,..,n$

$Set\ W = \{1\}; T = \{2,3,,..,n\}$

$Step2:$ Find $k \in T$ where $\lambda_k = \min_{j \in T} \lambda_j$

$Let\ N = \{l \mid \lambda_l \le \lambda_k + d\}$

$W = W \cup N$

$T = T \setminus N$

$Step3:$ For all $j \in T$, set $\lambda_j = \min\{\lambda_j, \lambda_l + c_{lj}\}$ for all $l \in N$

Go to step 2 unless $T = \varphi$. Even if you replace step 3 by Go to step 2 unless $T = \phi$, the algorithm will be correct.

- Problem 4: Assume that the shortest paths from node 1 to all other nodes are found and are have lengths λ_j, $j \neq 1$. If link *(i,k)* increases in length, paths that do not traverse link *(i,k)* are not affected by this change. Therefore, we can initialize Dijkstra's algorithm as below and proceed as in regular Dijkstra:

$P = \{j \mid$ shortest path from 1 *to* j does not traverse arc $(i,k)\}$

$\lambda_j = \lambda_j^*$ for all $j \in P$

$\lambda_j = \min_{l \in P}[\lambda_l^* + c_{lj}]$ for all $j \notin P$

- Problem 5: Trivial
- Problem 6: Dijkstra won't work. Floyd-Warshall and BMDP will work as long as there are no negative cycles. For acyclic, algorithm based on topological ordering is the best.
- Problem 7: Look at the reference given.