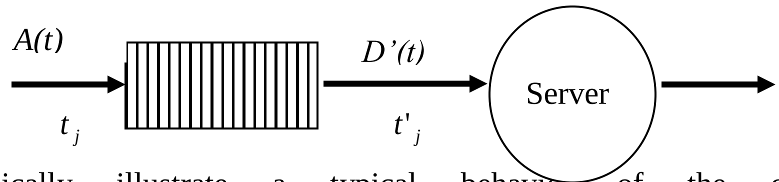
Dept. of ECE **Spring 2004**
Univ. of Conn. **KRP**

<u>Homework Set # 1 (Due January 28, 2004)</u>

1. Consider the queuing system shown below. Suppose the departure process is counted as one corresponding to customer entry into service. Define the queuing process $Q_W(t) = A(t) - D'(t)$, where $Q_W(t)$ is the number of customers who have arrived since time $t = 0$ but have not yet left the queue.



a) Graphically illustrate a typical behavior of the counting processes $A(t), D'(t)$ and $Q_W(t)$.

b) Derive Little's theorem relating the average queue length Q_W and the average waiting time W assuming a first-come first-served (FCFS) queuing discipline.

2. Little's theorem is valid for any work conserving queuing discipline.

a) Derive Little's theorem relating the average response time and average system queue length assuming a last-come first-served (LCFS) queuing discipline.

b) Show that Little's theorem is valid for systems where the order of customer service arbitrary, and where customer service can be interrupted to serve customers of higher priority.

Hint: You may want to refer to the following *classic* papers on Little's formula.

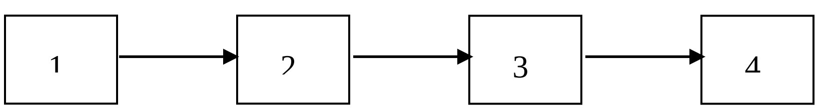
i) Little, J.D.C., "A Proof of the Queuing Formula $L = \lambda W$," <u>Operations Research</u>, Vol. 9, 1961, pp. 383-387.

ii) Jewell, W.S., "A Simple Proof of $L = \lambda W$," <u>Operations Research</u>, Vol. 15, 1967, pp. 1109-1116.

iii) Eilon, S., "A Simpler Proof of $L = \lambda W$," <u>Operations Research</u>, Vol. 17, 1969, pp. 915-916.

iv) Stidham, S., Jr., "A Last Word on $L = \lambda W$," <u>Operations Research</u>, Vol. 22, 1974, pp. 417-421.

3. Four processors are arranged in a pipeline to execute a stream of jobs.



Four Stage Processor Pipeline

Six jobs are executed on this system, and require the following processing at each stage:

a) Construct a schedule for executing the six jobs on this processor pipeline.

b) Compute the average response time for the above schedule:

Table I: Processing Times per Stage

Job	Stage 1	Stage 2	Stage 3	Stage 4
1	4	4	5	4
2	2	5	8	2
3	3	6	7	4
4	1	7	5	3
5	4	4	5	3
6	2	5	5	1

$$R = \frac{1}{N}\sum_{k=1}^{N} R_k;\ N = 6$$

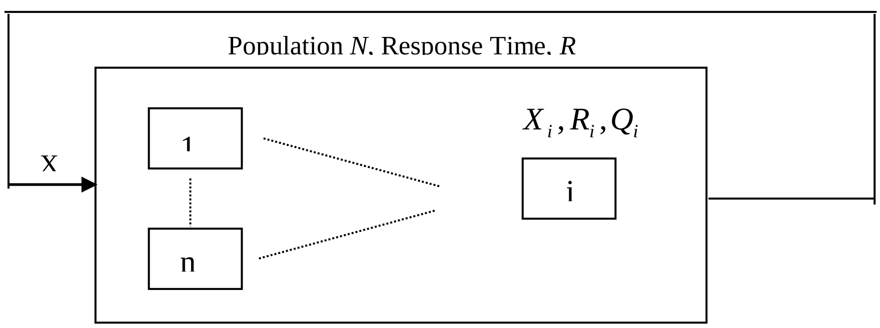
c) Let Q*(t)* denote the number of jobs in the system, either waiting to be executed or in execution, at time *t*. Calculate the average number of jobs in the system, Q:

$$Q = \frac{1}{T_F}\int_0^{T_F} Q(t)dt$$

d) Show that

$$Q = \frac{N}{T_F}R = XR;\ X = Average\,Throughput$$

4.



Consider the closed queuing network with n subsystems and population N shown above. Use Little's formula to show that:

$$R = \sum_{i=1}^{n} \frac{X_i}{X} R_i$$

(Hint: Use the fact that the queue length of the entire network $Q = N, Q = \sum_{i=1}^{n} Q_i$).

5. Suppose in the network of problem 4, there are J different classes of customers with populations $\{N_j\}_{i=1}^{J}$. Suppose the network throughput and network response time of each customer class is $\{X_j, R_j\}_{j=1}^{J}$. Use Little's formula to show that the overall network response time (for a customer selected at random) is given by:

$$R = \frac{\sum_{j=1}^{J} X_j R_j}{\sum_{j=1}^{J} X_j} = \frac{\sum_{j=1}^{J} N_j}{\sum_{j=1}^{J} X_j}$$

6. Messages are processed by a transmitter and then a receiver. The order of processing messages at the transmitter and receiver is identical. The processing time for message *i* by the transmitter is denoted by T_i, and by the receiver is denoted by R_i. There are a total of *N* messages to be transmitted at time zero. We define $T_{N+1} = R_0 = 0$ for notational simplicity. The receiver can buffer at most two messages at any one time; once the receiver has two messages, the transmitter stops processing messages until the receiver only has one message.

a) What are the precedence relations for processing *N* messages?

b) Show that the total time or make span to process all *N* messages can be written as

$$make\ span = T_F = \sum_{i=0}^{N} \max[T_{i+1}, R_i]$$

c) Show that the total time or make span to process all *N* messages can be written as

$$make\ span = T_F = \sum_{i=1}^{N} (T_i + R_i) - \sum_{i=1}^{N-1} \min[T_{i+1}, R_i]$$

d) The average throughput is defined as:

$$Average\,Throughput = \lim_{N \to \infty} \frac{N}{T_F}$$

Show that

$$Average\,Throughput = \frac{1}{E[\max(T,R)]} = [\lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N} \max[T_{i+1}, R_i]]^{-1}$$

where $E[\max(T,R)]$ is the average of the maximum of the transmitter and receiver time per message. For $T_k = T = constant, R_k = R = constant,$ explicitly evaluate this expression. For *R=1* and *T=1, 0.5, 0.2,* what is the mean throughput?

e) What changes if the receiver can buffer an infinite number of messages? (Hint: Read about Johnson's rule in the scheduling literature. See [PIN95], [BAK74] or [CON67]).

7. A single processor computer system executes two different classes of jobs from two different users. Assume that the think time and service times of the two job types at the node are as follows:

   Job class 1: $Z_1 = 100\,\text{sec.}, t_1 = 100\,\text{sec.}$

   Job class 2: $Z_2 = 600\,\text{sec.}, t_2 = 400\,\text{sec.}$

   Using the asymptotic bounding analysis, sketch the allowable throughput in the $X_1 - X_2$ plane, where $\{X_j\}_{j=1}^{2}$ is the throughput of job class *j, j=1,2.* (Hint: define the states of the system as: (1) both users in think mode, (2) user 2 in think mode and user 1's request being processed, (3) user 1 in think mode and user's 2 request being processed, (4) user 1's request waiting and user 2's request being processed, and (5) user 2's request waiting and user 1's request being processed. Find linear inequalities linking $Z_1, Z_2, t_1, t_2, X_1, X_2$ to the state probabilities $\{\pi(k)\}_{k=1}^{5}$ and plot). See [STU85].