Dept. of ECE **Spring 2004**
Univ. of Conn. **KRP**

Homework Set # 4 (Due March 3, 2004)

1. Consider an M|M|1 system with parameters λ, μ in which the customers are impatient. Specifically, upon arrival, customers estimate their queuing time w and then join the queue with probability $e^{-\alpha w}; \alpha \geq 0$ (or leave with probability 1-$e^{-\alpha w}$). The estimate is $w = \frac{n}{\mu}$, when the new arrival finds *n* in the system.
   a. Draw the state transition diagram and write the global balance equations. Give an expression for steady-state p_0 in terms of the system parameters.
   b. For $\alpha \to \infty$, find p_n explicitly and find the average number in the system.
2. Consider a birth-death queuing system in which $\lambda(n) = (n+2)\lambda$ and $\mu(n) = n\mu$. Solve for steady-state $\{p_n\}$ in terms of λ, μ *and* n only. Also, find the average system length.
3. Consider an M|M|m system that is to serve the pooled sum of two Poisson arrival streams; the *i^th^* stream has an average arrival rate given by λ_i and exponentially distributed service times with mean $1/\mu_i$ (*i=1,2*). The first stream is an ordinary stream with whereby each arrival requires exactly one of the *m* servers; if all *m* servers are busy then any newly arriving customer of type 1 is lost. Customers from the second class each require the simultaneous use of m_0 servers 9 (and will occupy them all simultaneously for the same exponentially distributed amount of time whose mean is $1/\mu_2$ seconds); if a customer from this class finds less than m_0 idle servers, then he too is lost to the system. Find the fraction of type 1 customers and type 2 the fraction of type 2 customers that are lost.
4. Show that the waiting time distribution for a FCFS queuing discipline at an M|M|1 queue, $F_W(t) = P(W \leq t) = 1 - \rho \exp[-\mu(1-\rho)t]$. What is the density $f_W(t)$?
5. Suppose that the birth-death coefficients are as follows: $\lambda(n) = \lambda/(n+1)$ and $\mu(n) = \mu.$ Show that the steady-state distribution $\{p_n\}$ takes the same form as that of M|M|∞ system. Find the throughput.
6. Consider a system that is identical to M|M|1 except that when the system empties out, service does not begin again until *K* customers are present in the system (*K* is given). Once service begins, it proceeds normally until the system becomes empty again. Find the steady-state probabilities of the number in the system, the system length, and the average delay per customer (Hint: the state description for the number of customers in the system *1,2,…,K-1* should include the service mode; waiting for customers to exceed the threshold or serving customers proceeds normally).
7. Derive the waiting time and response time distributions, $F_W(t)$ *and* $F_R(t)$, for M|M|1 queue.

8. Consider an M|M|1 queue in which the server has an exponentially distributed life-time with mean $1/f$. Once failed, the server is repaired; such a repair takes an exponentially distributed amount of time, with mean $1/r$. Customer arrivals from a Poisson process with rate λ and services last a negative exponentially distributed time with mean $1/\mu$. For the time being, assume that the arrival process is stopped as soon as the server breaks down.
   a. Draw the state-transition diagram for this model and derive the global balance equations.
   b. Derive a formula for the average system length.
   c. Now assume that arrivals continue to occur, even if the server has broken down. How does this change the state-transition diagram and the global balance equations? What is the stability condition in this situation?

   (Hint: Read section 4.6 of [VIS94] and references cited in section 4.15 on queues with breakdowns).

9. Consider the time-shared computer system model consisting of a terminal node and the computer system node with population *N*. The recursive MVA equations for the queue length, response time, and the throughput were derived in class as:

$$R(n) = \frac{1}{\mu}[1 + Q(n-1)]; Q(0) = 0 \qquad (1)$$

$$X(n) = \frac{n}{R(n) + \lambda^{-1}} \qquad (2)$$

$$Q(n) = X(n)R(n) \qquad (3)$$

   where *Q(n), R(n),* and *X(n)* are the queue length, response time, and the throughput with population n, λ^{-1} is the mean think time, and μ is the service rate of the computer system. The problem with recursive MVA equations (1)-(3) is that the performance measures for all intermediate populations $n=1,2,\ldots,N-1$ must be obtained before the performance measures for population *N* can be obtained. To avoid this complexity, the following approximation is proposed. Assume that the queue length is proportional to population, i.e., $\frac{Q(n-1)}{n-1} = \frac{Q(n)}{n}$. Using this approximation for $n=N$, show that *X(N)* satisfies the quadratic equation: $(N-1)X^2(N) - N(N\lambda + \mu)X(N) + \lambda\mu N^2 = 0$. (Hint: use the fact that $Q(N) + \frac{X(N)}{\lambda} = N$). Solve several problems using this approximation. What is the maximum relative error in throughput?