Dept. of ECE **Spring 2004**
Univ. of Conn. **KRP**

<u>Homework Set # 6 (Due April 14, 2004)</u>

The purpose of this HW set is to explore a wide variety of applications of what you learned.

1. Consider a scenario wherein N tasks compete for human's attention. Each task has inter-arrival time (based on system dynamics) that is exponentially distributed with mean $1/\lambda$ seconds. Assume that the service time per request is also *grossly approximated* as being exponentially distributed with mean $1/\mu$ seconds. The service time is computed from one of the following four modes of human interaction with the system.

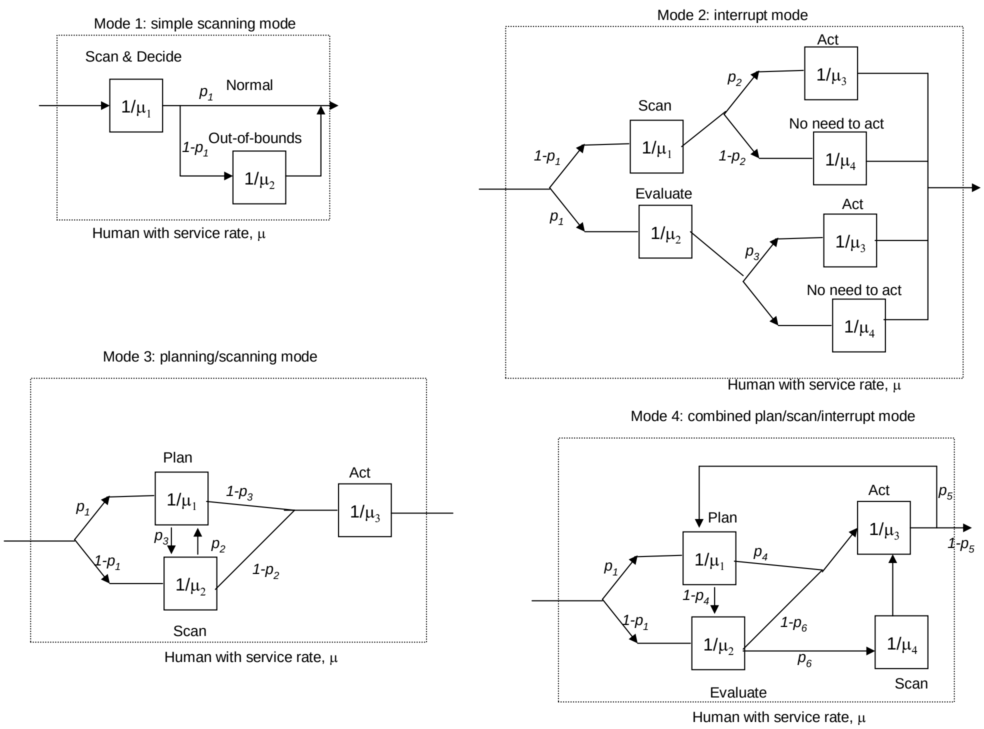


Fig. 1: Human attention models

a) For each mode of operation, compute the effective mean service time, $1/\mu$.

b) Assume the following parameter values for each mode: $1/\lambda$ = 5 seconds.

Mode 1: $1/\mu_1 =$ 0.5 seconds; p_1 =0.9 and $1/\mu_2 =$ 3 seconds

Mode 2: $1/\mu_1 =$ 0.5 seconds; p_1 =0.1; p_2 =0.1; p_3 =0.5; $1/\mu_2 =$ 2 seconds; $1/\mu_3 =$ 3 seconds and $1/\mu_4 =$ 0.4 seconds

Mode 3: p_1 =0.5; $1/\mu_1 =$ 2 seconds; $1/\mu_2 =$ 0.5 seconds; p_3 =0.5; p_2 =0.2; $1/\mu_3 =$ 3 seconds

Mode 4: p_1 =0.5; $1/\mu_1 =$ 2 seconds; $1/\mu_3 =$ 4 seconds; p_4 =0.7; p_6 =0.5;

$P_5 = 0.2$; $1/\mu_4 = 0.5$ seconds

Determine the throughput, X; queue length, Q_h at the human; the response time, R_h, and the utilization, U_h for $N=1,2,..,10$. Compare solutions based on exact MVA, Schweitzer-Bard MVA heuristic and Chandy-Neuse Linearizer heuristic.

c) (bonus) Suppsoe that the service time is treated as series-parallel stages. Can you compute the measures of interest (Hint: you may want to read Chapter II of Jaiswal, "Priority Queues," Academic Press, NY, 1967 to solve this part of the problem.)

2. Consider the following manufacturing system consisting of 6 machine groups as shown in Fig. 2. The system produces a single part type with the following relative service times at each machine group. All processing times are exponentially distributed:

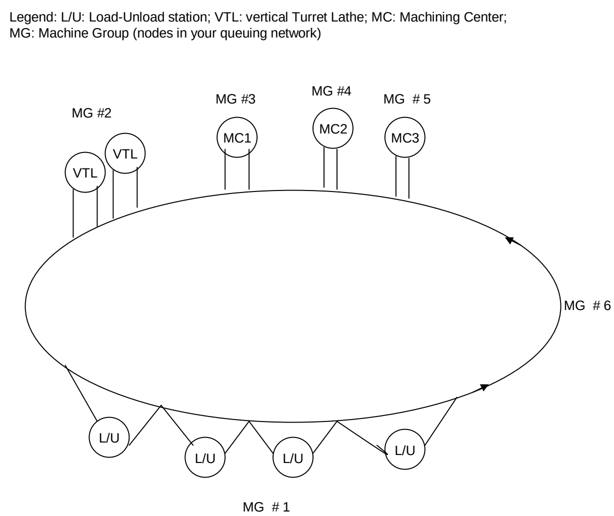


Figure 2: Manufacturing System

The processing times at the machine groups are as follows:

Machine	1	2	3	4	5	6
Processing Time in Minutes	15	10	10	10	5	2.5

a) Compute the throughput rate, response time, queue length and utilization at each nodeas a function of number of pallets (number of customers in the system).

b) Also, determine the probability distribution at each node

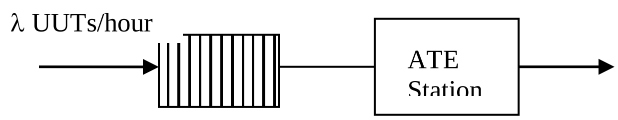
3. Consider a central server model with two I/O channels with respective rates $\mu_2 = 5$ sec^{-1} and $\mu_3 = 3$ sec^{-1}. The CPU service rate is $\mu_1 = 7$ sec^{-1}. The branching probabilities are $p_1 = 0.05$, $p_2 = 0.65$ and $p_3 = 0.30$. If the degree of multi-

programming, N is fixed, then the given network has product form. In practice, however, the degree of multi-programming is a random variable with mean $E(N)$. For $E(N)$ = 1,2,.., 10, find the throughput for the following problems:

a) N is deterministic, i.e., $E(N)=N$
b) N takes on two different values: $N=E(N)-1$ with probability 0.5 and $N=E(N)+1$ with probability 0.5.
c) N is Poisson distributed with mean $E(N)$
d) N is binomially distributed with parameters $n=2E(N)$ and p =0.5 so that $E(N)=np$.
e) N has a discrete uniform density over $\{1,2,\ldots, 2E(N)\}$
f) N takes on two values: N = 0 with probability 0.5 and N = $2E(N)$ with probability 0.5.

(Hint: Use $X = \sum_{allN} X(N)p(N)$).

4. Consider an Automatic Test System (ATE) as shown in Figure 3. An ATE designer is interested in evaluating three design concepts. The Unit Under Test (UUT) testing consists of three steps: hookup, test and fault isolation, and teardown.



a) <u>Concept 1:</u> In this concept, the ATE is represented as a single server vnode (called a single port station). The total service time is 1.8 hours (hookup and tear down = 0.4 hours, test and fault isolation = 1.4 hours).
b) <u>Concept 2:</u> In this concept, the ATE system is represented as a two node closed queuing network consisting of an infinite server node (representing ample technicians performing hookup and teardown) and a single server node performing test and fault isolation. As before, hookup and tear down = 0.4 hours, test and fault isolation = 1.4 hours. (Hint: use flow equivalent node concept to find effective service rates for various populations in the ATE system).
c) <u>Concept 3</u>: In this concept, the ATE system is represented as a nine node closed queuing network. The first node is an infinite server node representing hookup and tear down, and the remaining eight nodes are single server nodes representing test steps. The relative service times at each node are as follows:

node	1	2	3	4	5	6	7	8	9
Service time in hours	0.4	0.35	0.283	0.25	0.20	0.167	0.083	0.05	0.0167

(i) Compare the throughput capacity of each concept as aq function of the number of ports (i.e., population).

(ii) Compute the waiting and response times for λ=0.385 units/hour, λ=0.5 units/hour, and λ=0.6 units/hour.

5. Consider a two class product-form mixed network with one open class and one closed class. Derive recursive MVA algorithm for this mixed network. (Hint: See the notes and the book by Lavenberg).