

Homework Set # 7 (Due April 28, 2004)

1. Consider an M|G|1 system with the difference that each busy period is followed by a *single* vacation interval. Once this vacation is over, an arriving customer to an empty system starts service immediately. Assume that vacation intervals are independent, identically distributed, and independent of the customer inter-arrival and service times. Derive the average waiting time in queue.
2. *M|G|1 queue with overhead for each busy period.* Consider the M|G|1 queue with the difference that the service of the first customer in each busy period requires an increment (e.g., in setting up the machine) Δ over the ordinary service time of the customer. We assume that Δ has a given distribution and is independent of all other random variables in the model. Let $\rho = \lambda \bar{X}$ be the utilization factor. Show that:

a. $P_0 = P \{ \text{the system is empty} \} = \frac{(1 - \rho)}{(1 + \lambda \bar{\Delta})}$

b. Average length of the busy period = $\frac{(\bar{X} + \bar{\Delta})}{(1 - \rho)}$

c. The average waiting time in queue is: $W = \frac{\lambda \bar{X}^2}{2(1 - \rho)} + \frac{\lambda [(\bar{X} + \bar{\Delta})^2 - \bar{X}^2]}{2(1 + \lambda \bar{\Delta})}$

3. Problem 3.22 of Bertsekas and Gallager
4. Problem 3.23 of Bertsekas and Gallager
5. Problem 3.20 of Bertsekas and Gallager
6. Problem 4.4 of Bertsekas and Gallager
7. Problem 4.17 of Bertsekas and Gallager