The University of Connecticut **Fall 2012**
Dept. of ECE **KRP**

ECE 6095/4121
<u>Problem Set # 1</u>
(Due September 11, 2012)
(Homework and projects can be done in teams of at most three students. Use MATLAB when necessary)

1. Figure 1 below shows a block diagram of an automobile cruise control system, which is used to maintain the speed of a vehicle automatically at a constant level. The speed v of the car depends on the throttle opening u. The throttle opening is controlled by the cruise controller in such a way that the throttle opening is *increased* if the difference $(v_r - v)$ between the reference speed v_r and the actual speed is positive, and *decreased* if the difference is negative. This feedback mechanism is meant to correct automatically any deviations of the actual vehicle speed from the desired cruise speed.

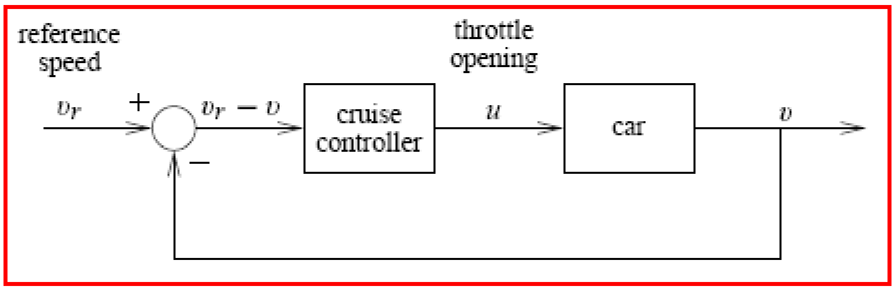

Fig. 1: Cruise Control System of an Automobile

Using Newton's law, $m\dot{v} = F(t) = cu(t) - \rho v^2(t)$ where m is the mass of the vehicle, the term $cu(t)$ represents the propulsion force of the engine, and is proportional to the throttle opening $u(t)$, with proportionality constant c. The throttle opening varies between 0 (shut) and 1 (fully open). The second term $v^2(t)$ is caused by air resistance. The friction force is proportional to the square of the speed of the car, with ρ as the friction coefficient. If $u(t) = 1, t \geq 0,$ the steady state velocity is $v_{ss} = \sqrt{c/\rho}$.

Define normalized velocity as $w(t) = v(t)/v_{ss} = v(t)\sqrt{\rho/c}$ and $r(t) = v_r(t)/v_{ss} = v(t)\sqrt{\rho/c}$

a) (Nonlinear System) *Show* that the differential equation for $w(t)$ is

$$\tau\dot{w}(t) = u(t) - w^2(t);\ \tau = \frac{m}{\sqrt{\rho c}}$$

$b)$ (*Explicit Solution*) *Show* that the solution to this equation for $u(t) = 1$ and $w(0) = 0$

is $w(t) = \tanh(\frac{t}{\tau}), t \geq 0.$

What would you select for τ if you want to go from 0 to v_{ss} in ≤ 8 sec?

$c)$ Develop a SIMULINKmodel of the system. Compare the solution with that in (b) for τ=3seconds.

d)(Linearization) To a constant throttle setting u_0 corresponds a steady-state cruise speed $w_0 = \sqrt{u_0}$. Let $u = u_0 + \delta u$ and $w = w_0 + \delta w$. Show that the *linearized* equation for the system is:

$$\delta \dot{w}(t) = -\frac{1}{\theta}\delta w(t) + \frac{\delta u(t)}{\tau};\, t \geq 0 \; with\; \theta = \frac{\tau}{2w_0} = \frac{\tau}{2\sqrt{u_0}}$$

e) (PI controller) Suppose there is a disturbance acting on the system (due to head/tail wind, uphill or down-hill g

That is, $\delta \dot{w}(t) = -\frac{1}{\theta}\delta w(t) + \frac{\delta u(t)}{\tau} + d(t)$

We want to use a proportional-integral (PI) control of the form

$$u(t) = u_0 + k_p(r(t) - w(t)) + k_I \int_0^t [r(\tau) - w(\tau)]d\tau.$$

(Why we didn't just use a proportional controller?). For $\tau = 3\sec$ *and* $w_0 = 0.5$, experiment with this controller to show that disturbance rejection and low-frequency disturbance attenuation may be achieved for the cruise control system with satisfactory gain and phase margins for any closed-loop bandwidth allowed by the system (recall that PI controller reduces the bandwidth).

f) (Evaluate Design) Show the Bode plots of the sensitivtiy and closed-loop transfer functions. Simulate the closed-loopsystem (with the controller above linked to the nonlinear model) in SIMULINK and show all the signals in the loop. Experiment with the system by adding high frequency sensor noise to $w(t)$.

2. Derive the equations of motion for the balance system (e.g., a human being balanced on a stabilizing cart) and verify that for small θ the dynamics are approximated by linearized dynamics. The inverted pendulum parameters are as follows:

M = Mass of base, 10kg

m = mass of pendulum, 80kg

l = Distance from the base to the center of mass to be balanced, 1m

J = Moment of intertia of system to be balanced, $100\, kg.m^2/s^2$

c = coefficients of viscous friction associated with base, 0.1kg/s=0.1N/m/s

γ = coefficient of friction associated with pendulum, 0.01 $kg.m^2/s$

Develop a Simulink model of the nonlinear system. Using the linearized dynamics, evaluate the poles of the linearized system. Is the system stable? Is the system controllable? Show that a controller of the form:

$$F(t) = -15.5r(t) - \begin{bmatrix} -15.6 & 1730 & -50.1 & 443 \end{bmatrix} \begin{bmatrix} p \\ \theta \\ \dot{p} \\ \dot{\theta} \end{bmatrix}$$

stabilizes the system with good response, but with a large input force. Can you design a better system that requires less input force and somewhat slower response. You can use *place* or *Acker* command to get the feedback gains and the gain on reference input r(t) to get unity closed-loop dc gain.

3. (Background) Given the armature-controlled DC motor parameters in Lecture 1, compute the numerical values of the open-loop transfer function, $G(s) = \dfrac{\theta(s)}{v_i(s)}$.

   Consider two feedback control schemes for the motor. In the first scheme, the input voltage to the motor is given by $v_i(t) = K\theta_r(t) - k_1\omega(t) - k_2\theta(t)$, where $\theta_r(t)$ is the reference angular position signal. Draw the signal flow diagram of the closed-loop system and compute the closed-loop transfer function $T(s) = \dfrac{\theta(s)}{\theta_r(s)}$. Select K, k_1, k_2 so that the DC gain of the system, $T(0)$, is unity, percent overshoot to a step input is <5% and settling time (for 1% error) is 0.25 seconds. Plot the step response and draw pole-zero configuration of the closed-loop system using *pzmap*.

   In the second scheme, the input voltage to the motor is given by

   $$\frac{1}{\beta\omega}\dot{v}_i + v_i = \frac{K}{\omega}\dot{e}(t) + Ke(t);\, e(t) = \theta_r(t) - \theta(t).$$

   Draw the signal flow diagram of the closed-loop system and compute the closed-loop transfer function $T(s) = \dfrac{\theta(s)}{\theta_r(s)}$. Select K, ω, β to satisfy the same requirements as in scheme 1. Draw pole-zero configuration of the closed-loop system using *pzmap*. Discuss the two designs.

Presentation Project: (**Due October 2, 2012**)

Each team will become knowledgeable about and explore in-depth a physical process or a system, a sensor and an actuator. Prepare three separate files of power point slides (a total of at least 45-60 slides) describing the system model, sensor(s) and actuator(s). For the system, be sure to include how it works, mathematical models, linearization (if needed), Simulink block diagram, simulation output and references. For the sensor and actuator, be sure to include their basic working principles, mathematical models, sample applications, specifications, vendors, advantages and limitations, economic factors (cost, reliability, etc.), and, of course references you used. You may eventually use this system model for your class project. So, select a system domain you really like.

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