The University of Connecticut **Spring 2016**
Dept. of ECE **KRP**

Homework Set # 1 (Due January 28, 2016)

1. Consider an m by m matrix B. Suppose that we want to replace the i^{th} column of B by another m vector $\underline{a}$. That is, we remove the i^{th} column $\underline{b}_i$ from B and put $\underline{a}$ in its place. Write this operation in matrix form. This will be useful in the revised simplex algorithm.

2. (a) Is the product of two convex functions convex? If yes, prove it. If not, give a counter example.
   (b) Using the fact that the Hessian of a strictly convex function is positive definite, determine whether or not the following functions are strictly convex in the indicated domains.
   (i) $f = x_1 x_2$, $X = R^2$; (ii) $f = exp\ (x_1 + x_2)$, $X = R^2$;
   (iii) $f = tan\ x_1$, $X = \{ x_1 : 0 < x_1 < 1\}$; (iv) $f = exp\ (- x_1 - x_2) + x_1^2 - 2\ x_1$
   (v) $f = max\ (f_1 , f_2)$ where $f_1 = x_1^2 + x_2^2$ and $f_2 = 2x_1^2 - x_2$

3. Let $f(\underline{x})$ be convex in R^n.
   (a) is $f\ (A\underline{x} + \underline{b})$, where A is an m *by* n matrix and $\underline{b}$ is an m vector, convex
   (b) Fix components $x_2, \ldots\ldots, x_n$. Consider $g\ (x_1) = f\ (x_1, x_2, \ldots\ldots, x_n)$. Is $g\ (x_1)$ convex?

4. Prove that $f(\underline{x}) = \underline{x}^T Q\ \underline{x}$ is convex, if Q is positive definite.

5. Suppose that $f_1, f_2, \ldots\ldots, f_n$ are convex functions from R^n into R and let $f\ (\underline{x}) = \sum_{i=1}^{n} f_i\ (\underline{x})$. Show that if each f_i is convex, so is f.

6. Show that a hyperplane $H = \{\underline{x} : \underline{c}^T \underline{x} = k\ \}$ and halfspace $H^+ = \{\ \underline{x} : \underline{c}^T \underline{x} \leq k\ \}$ are convex sets.

7. Exercise 2.1 of Text, Page 75.

8. Exercise 2.2 of Text, Page 76.

9. Suppose that we have the LU decomposition of an m by m matrix B. Suppose we replace column i of B (i.e., $\underline{b}_i$) by a new column $\underline{a}$ of the same dimension. Devise an $O(m^2)$ algorithm to find new L and U. This will be useful in efficient implementation of revised simplex algorithm. See Golub and Van Loan or a similar book on Matrix Computations.

10. Suppose that we have the QR decomposition of an m by m matrix B. Suppose we replace column i of B (i.e., $\underline{b}_i$) by a new column $\underline{a}$ of same dimension. Devise an $O(m^2)$ algorithm to find new Q and R. This will be useful in the efficient implementation of revised simplex algorithm. See Golub and Van Loan or a similar book on Matrix Computations.