Problem Set # 10

UNSYMMETRIC EIGEN VALUE PROBLEM

(Due Nov 12, 2008)

(Do only Problems 1 and 5)

1. Suppose that the power method is applied to:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

with $\underline{u}^0 = [1\ 1]^T$. What is $\lambda^{(20)}$ *and* $\underline{u}^{(20)}$.

2. Show that the following iteration provides the p smallest Eigen values of A:

    For k=1,2,…. Do until Z_k converge

        Solve $AZ_k = Q_{k-1}$

        $Q_k R_k = Z_k$ (QR decomposition of Z_k)

    End

    where Q_0 is an n x p orthonormal matrix and Z_k is an n x p matrix.

3. Show that if A is an n x n matrix and $\underline{z}$ is an n vector, then there exists an orthogonal matrix Q such that $Q^T AQ$ is upper Hessenberg and $Q\underline{z}$ is a multiple of $\underline{e}_1 = [1\ 0\ 0\ \ldots 0]^T$.

4. Explain how single shift QR step $A_0 - \mu I = QR$ and $A_1 = RQ + \mu I$ can be carried out implicitly. That is, show how the transition from an upper Hessenberg A_0 to another upper Hessenberg A_1 can be carried out without subtracting the shift explicitly from the diagonals of A_0.

5. Consider a j step QR of the following form:

$$A_k - \mu_k I = Q_k R_k,\ A_{k+1} = R_k Q_k + \mu_k I;\ A_0 \text{ is upper Hessenberg.}$$

Show that: $(Q_1\ Q_2\ \ldots\ Q_J)\ (R_J\ R_{J-1}\ \ldots.\ R_1) = (A_0 - \mu_1 I) \ldots. (A_0 - \mu_J\ I)$

Computational: Examine the use of eig in MATLAB to find the Eigen values of an n x n matrix A. Select test examples with elements of A widely varying and investigate if balancing has any effect on the solutions.