The University of Connecticut
Dept. of ECE

Fall 2008
KRP

Problem Set #12
Singular Value Decomposition
(Due Nov 19, 2008)

1)

(a) Show that if $(A^T A + \lambda I)\underline{x} = A^T \underline{b}$, $\lambda > 0$, and $\|\underline{x}\|_2 = \alpha$, then $\underline{z} = (A\underline{x} - \underline{b})/\lambda$ solves the *dual equations* $(AA^T + \lambda I)\underline{z} = -\underline{b}$, $\|A^T \underline{z}\|_2 = \alpha$.

(b) Show that if $(AA^T + \lambda I)\underline{z} = -\underline{b}$, $\|A^T \underline{z}\|_2 = \alpha$, then $\underline{x} = -A^T \underline{z}$ satisfies $(A^T A + \lambda I)\underline{x} = A^T \underline{b}$, $\|\underline{x}\|_2 = \alpha$.

2) Suppose $A \in R^{m x n}$ and that $\|\underline{u}^T A\|_2 = \sigma$ with $\underline{u}^T \underline{u} = 1$. Show that if $\underline{u}^T (A\underline{x} - \underline{b}) = \underline{0}$ for $\underline{x} \in R^n$ and $\underline{b} \in R^m$, then $\|\underline{x}\|_2 \geq |\underline{u}^T \underline{b}| / \sigma$.

3) Show that if A and B are m by p matrices, with $p \leq m$, then

$$\min_{Q^T Q = I_p} \|A - BQ\|_F^2 = \sum_{i=1}^{p} (\sigma_i^2(A) - 2\sigma_i(B^T A) + \sigma_i^2(B))$$

4) For the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

find $R(A)$, $N(A)$, $R(A^T)$ and $N(A^T)$ using SVD.