ECE 336 KRP
Dept. of ECE Fall 2008

Problem Set #14
Riccati Equation

(Do only problems 4 and 6)

1. Solve the algebraic Riccati equation: $A^T P + PA + CC^T - PBB^T P = 0$ with the following parameters (A,B,C):

$$A = \begin{bmatrix} -4 & -2 & 0 \\ -1 & -4 & -1 \\ -1 & -1 & -4 \end{bmatrix}; \; C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \; B = \begin{bmatrix} -1 & 2 \\ 1 & -2 \\ 1 & -2 \end{bmatrix}$$

2. Solve Problem 1 when:

$$B = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$$

3. Solve the Riccati equation $A^T P + PA + Q - PBR^{-1}B^T P = 0$ when:

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 0 & 3 \\ -1 & -2 & 1 \end{bmatrix}; \; B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}; \; R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. A 30 MW generator connected to an infinite bus bar is modeled by:

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -64.3 & -89 & -79.1 \\ -0.756 & 0 & -0.885 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 0.166 \end{bmatrix} u$$

Solve the optimal linear regulator problem when $R = 1$ and

$$Q = 10^4 \times \begin{bmatrix} 3.901 & 5.915 & 3.703 \\ 5.915 & 8.97 & 5.798 \\ 3.703 & 5.798 & 10.622 \end{bmatrix}$$

Plot the root locus of the closed-loop poles as R is varied.

5. One approach to designing controllers that are insensitive to plant parameter variations is to append sensitivity related terms to the quadratic objective function. Consider a scalar parameter α that is subject to variation. In this approach to insensitive controller design, we optimize:

$$J = \frac{1}{2}\int_0^\infty \left[\underline{x}^T(t)Q_1\underline{x}(t) + \beta \underline{y}^T(t)Q_2\underline{y}(t) + \underline{u}^T(t)R\underline{u}(t) \right] dt$$

where $\underline{y}(t) = {d\underline{x}(t)}/{d\alpha}$.

(a) Develop a procedure for designing a feedback control law of the form $\underline{u}(t) = -L_1\underline{x}(t) - L_2\underline{y}(t)$ to minimize the above cost function.

(b) A major problem with the control law in (a) is the need for computing or estimating $\underline{y}(t)$. A practical approach is to design a control law of the form $\underline{u}(t) = -L\underline{x}(t)$ to minimize the above cost function. Develop a procedure for finding the optimal gains L^*.

(c) Apply the design procedures to the following scalar problem:

$$\dot{x} = -\alpha^2 x + u$$

with $x(0) = 1$ and he nominal value of $\alpha = 1$. Assume $B = 1, Q = 1,$ and $R = 1$. Compare your designs to the case when $\beta = 0$.

6. Design a linear quadratic regulator for the plant given by:

$$\dot{\underline{x}} = \begin{bmatrix} -0.074 & 1 & -0.012 \\ -8.0 & -0.055 & -6.2 \\ 0 & 0 & -6.67 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 6.67 \end{bmatrix} u$$

Assume that $Q = Diag(q_1, q_2)$ and $R = 1$. Perform your design for the following three cases:

(i) $q_1 = 10,\ q_2 = 0.5$
(ii) $q_1 = 1000,\ q_2 = 2.0$
(iii) $q_1 = 10^7,\ q_2 = 2000$

Which design would you prefer and why?