The University of Connecticut **Spring 2016**
Dept. of ECE **KRP**

Homework Set # 2 (Due Friday February 5, 2016)

1. A scientist has observed a certain quantity Q as a function of a variable t. He is interested in determining the mathematical relationship relating t and Q, which assumes the following polynomial form:

$$Q(t) = \sum_{i=0}^{n-1} a_i t^i;\ a_i \geq 0\,; \sum_{i=0}^{n-1} a_i = 1$$

From the observed results of his n experiments $(t_1, Q_1), (t_2, Q_2), \ldots\ldots, (t_m, Q_m), m > n,$ he wants to determine the coefficients {a_i} such that a measure of the error between the observed values and those predicted by the polynomial function is minimized. Consider the following three measures of error:

$$(i) Minimize\, f = \sum_{i=1}^{m} | Q_i - Q(t_i) | \Rightarrow 1 - norm\ of\ error$$

$$(ii) Minimze\, f = \max_i | Q_i - Q(t_i) | \Rightarrow \infty - norm\ of\ error$$

$$(iii) Minimize\, f = \sum_{i=1}^{m} (Q_i - Q(t_i))^2 \Rightarrow square\ of\ 2 - norm$$

Show that the scientist's problem reduces to a linear programming problem under criteria (i) and (ii) and to a quadratic programming (least squares) problem under criterion (iii).

2. Convert the following LP problem to standard form:

$$Maximize\ x_1 + 4x_2 + x_3$$

$$\begin{aligned} subject\ to\ 2x_1 - 2x_2 + x_3 &= 4 \\ x_1 - x_3 &= 1 \\ x_2 \geq 0, x_3 &\geq 0 \end{aligned}$$

Find the optimal solution using Excel Solver (or) MATLAB linprog and by hand.

3. An oil refinery has two sources of crude oil: a light crude that costs \$56 per barrel and a heavy crude that costs \$50 per barrel. The refinery produces gasoline, heating oil, and jet fuel from the crude in the amounts per barrel indicated in the following table:

	Gasoline	Heating Oil	Jet Fuel
Light Crude	0.3	0.2	0.3
Heavy Crude	0.3	0.4	0.2

The refinery has contracted to supply 900,000 barrels of gasoline, 800,000 barrels of heating oil, and 500,000 barrels of jet fuel. The refinery wishes to find the amounts of light and heavy crude to buy so as to be able to meet its obligations at minimum cost. Formulate this problem as a linear program, and solve for the optimal solution.

4. A small computer manufacturing firm forecasts the demand over the next n months to be d_i, i=1,2.., n. In any month, it can produce r units using **regular** production at a cost of b dollars per unit. By using **overtime**, it can produce additional units at c dollars per unit, where c > b. The firm can store units form month to month at a cost of s dollars per unit per month. Formulate the problem of determining the production schedule that minimizes cost (Hint: Formulate the problem as a minimization of a piece-wise linear function).

5. Exercise 1.4 of Text. Solve using Excel Solver or MATLAB linprog and by hand.

6. Exercise 1.8 of Text

7. Exercise 1.13 of Text

8. Exercise 1.15 of Text

9. Support vector machines (SVM) are popular techniques in data-driven classification and regression. A variant of SVM classifier (1-SVM) that employs the 1-norm of the weight vector has the following objective function:

$$f(\underline{w}, w_0) = \sum_{n=1}^{N} \max(0.,1 - y_n t_n) + \lambda \| \underline{w} \|_1$$

where $y_n = \underline{w}^T \phi(\underline{x}_n) + w_0$ and $\{\phi(\underline{x}_n)\}_{n=1}^{N}$ are known. Show that this problem can be formulated as a linear programming problem.