The University of Connecticut **Fall 2012**
Dept. of ECE **KRP**

ECE 41421 and 6095
<u>Problem Set # 3</u>
(Due October 2, 2012)

1. Use the ***c2d*** function to compute $\Phi = e^{Ah}$ and $\Psi = \int_0^h e^{A\sigma} d\sigma$ to find the equivalent discrete system $\underline{x}(k+1) = \Phi\underline{x}(k) + \Gamma u(k),\ y(k) = C\underline{x}(k)$ for the continuous system models $\dot{\underline{x}}(t) = A\underline{x}(t) + Bu(t),\ y(t) = C\underline{x}(t)$:

(i) $A = \begin{bmatrix} 1.2 & 0 & 0.5 \\ 0.5 & -0.2 & 0.5 \\ 0.8 & 2.1 & -1.3 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ $C = \begin{bmatrix} -1.2 & 2 & 1.5 \end{bmatrix}$ with $h = 0.3$ sec.

and

(ii) $A = \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -0.5 \\ 1.2 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 1.5 \end{bmatrix}$ with $h = 0.5$ sec.

(a) Obtain $\tilde{G}(z)$ for both cases using ***ss2tf*** function.

(b) ANALYTICALLY verify the results of $\tilde{G}(z)$ for system (ii) only. Compute $\tilde{G}(z)$ using both methods, i.e., via the discrete state equations, and directly from G(s). Note in the latter approach, it will be judicious to use the partial fraction expansion of G(s).

(c) Do you think the suggested value for h is OK or too big or too small for use in the digital control of the above system (i)? Explain.

2. (a) Show the Bode plot of G(s) with $s = j\omega$ for both systems in problem 1.

(b) Show the Bode plot of $\tilde{G}(z)$ with $z = e^{j\omega h}$ for these systems.

(c) Over $\omega \in \left[0, \frac{\pi}{h}\right]$ how big is the difference between $\tilde{G}(e^{j\omega h})$ and $G(j\omega)e^{-\frac{j\omega h}{2}}$? Note that MATLAB's Bode function allows you to specify '**iodelay**'.

3. Compute Γ_1 and Γ_2 for the system in Prob. 1-(ii) with a delay $\in = 0.2$ sec. Compare the Bode plot of $\tilde{G}(z)$ with $\in = 0.2$ sec with that of $\in = 0$.

4. (a) For a system with sample time h, what is the equivalent discrete system model $\tilde{G}(z)$ for a pure time delay $G(s) = e^{-sh}$?

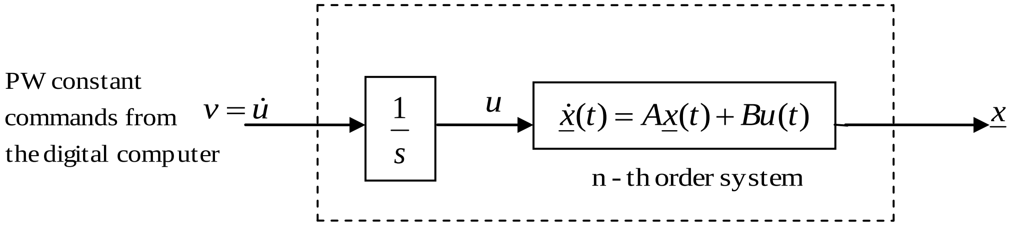
(b) Why is the equivalent discrete system for a delay of $h/2$ sec $\neq z^{-1/2}$?

(c) Find an equivalent discrete system for a pure time-delay of $h/2$ sec in the (first-order) form $\tilde{G}(z) = (az+b)(cz+d)^{-1}$ by dividing the $\tilde{G}(z)$ for the system

$G(s) = \dfrac{e^{-sh/2}}{(s+1)}$ with that of $G(s) = \dfrac{1}{s+1}$.

(d) Compare Bode plots of your $\tilde{G}(z)$ and $e^{-sh/2}$.

5. Often in control design, the "input" is not $u(t)$ but $\dot{u}(t)$ as shown below:



Consider the (n + 1)th order equivalent discrete system model for the above case in the form $\underline{x}(k+1) = \Phi_a \underline{x}(k) + \Gamma_a v(k)$ where $\underline{x}(k) = \begin{bmatrix} \underline{x}(k) \\ u(k) \end{bmatrix}$

and $v(k)$ is the PW constant "input". Note that the resulting $u(t)$ is PW linear.

(a) Show that

$$\Phi_a = \begin{bmatrix} \Phi & \Gamma \\ \underline{0}' & 1 \end{bmatrix}$$ where Φ and Γ are the usual matrices that are associated with A and B

(b) What is Γ_a?

(c) What is the above discrete model for $\dot{\underline{x}}(t) = -1.4\underline{x}(t) + 0.5u(t)$ for $h = 0.25$?