The University of Connecticut **Spring 2016**
Dept. of ESE **KRP**

Homework Set # 3 (Due February 18, 2016)

1. A large textile firm has two manufacturing plants, two sources of raw material, and three market centers. The transportation costs between the sources and the plants and between the plants and the markets are as follows:

Table I: Source - Plant Transportation Costs

	Plant A	Plant B
Source 1	\$2/ton	\$3/ton
Source 2	\$4/ton	\$3/ton

Table II: Plant - Market Transportation Costs

	Market 1	Market 2	Market 3
Plant A	\$8/ton	\$4/ton	\$2/ton
Plant B	\$6/ton	\$8/ton	\$4/ton

Ten tons are available from source 1 and 15 tons from source 2. The three market centers require 8 tons, 14 tons and 3 tons, respectively. The plants have unlimited processing capacity.

a) Formulate the problem of finding the shipping patterns from sources to plants to markets that minimizes the total transportation cost. Solve using MATLAB linprog.

b) Reduce the problem to a single standard transportation problem with two sources and three destinations (i.e., markets). (Hint: Find the minimum cost paths from sources to markets).

c) Suppose that plant A has a processing capacity of 8 tons, and plant B has a processing capacity of 17 tons. Show how to reduce the problem to two standard transportation problems.

2. A machine shop has one drill and five milling machines, which are to be used to produce an assembly consisting of parts 1 and 2. The productivity of each machine for the two parts is given below:

Table III: Production Time in Minutes/ Piece

	Drill	Mill
Part 1	3	20
Part 2	5	15

It is desired to maintain a balanced loading on all machines such that no machine runs more than 30 minutes per day longer than any other machine (assume that the milling load is split evenly among all five milling machines.) Divide the work time of each machine to obtain the maximum number of completed assemblies, assuming an 8-hour working day.

3. In many applications of linear programming, it may be sufficient, for practical purposes, to obtain a solution for which the value of the objective function is within a predetermined tolerance ε from the minimum value f^*. Stopping the simplex algorithm at such a solution rather than searching for the true minimum may considerably reduce the computations. Consider a linear programming problem for which the sum of the variables is known to be bounded above by s. Let f_0 denote the current value of the objective function at some stage of the simplex algorithm, p_j the corresponding relative cost coefficient, and $M = \max_j p_j$. Show that if $M = \varepsilon/s$, then $f_0 - f^* \leq \varepsilon$.

4. Rather than select the variable corresponding to the most negative relative cost coefficient as the variable to enter the basis, it has been suggested that a better criterion would be to select that variable which, when pivoted in, will produce the greatest improvement in the objective function. Show that this criterion leads to selecting the variable x_{Nk} corresponding to the index k minimizing $\max_{i:\alpha_{ik}>0} \frac{p_k \beta_i}{\alpha_{ik}}$.

5. A company produces refrigerators, stoves, and dishwashers. During the coming year, demand for these products is as follows:

Table I: Quarterly Demand for Products

Product	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Refrigerators	1500	1000	2000	1200
Stoves	1500	1500	1200	1500
Dish Washers	1000	2000	1500	2500

The company wants a production schedule that meets the demand requirements. The management has also decided that inventory level for each product must be at least 150 units at the end of each quarter. There is no inventory of any product at the start of the first quarter. Assume that each item left in the inventory at the end of a quarter incurs a holding cost of $5. During a quarter only 18000 hours of production time are available. A refrigerator requires 2 hours, a stove 4 hours, and a dishwasher 3 hours of production time. Refrigerators cannot be manufactured in the fourth quarter because the company plans to modify tooling for a new product line. Find a production schedule over the year to meet the quarterly demands and minimizes the inventory cost. Formulate the problem as an LP and solve it using the revised simplex algorithm. You can use Solver or an equivalent.

6. Solve using the revised simplex solver and using Dantzig-Wolfe decomposition method:

$$\begin{array}{lrl}
\text{minimize} & -4x_1 - x_2 - 3x_3 - 2x_4 & \\
\text{subject to} & 2x_1 + 2x_2 + x_3 + 2x_4 & \leq 6 \\
 & x_2 + 2x_3 + 3x_4 & \leq 4 \\
 & 2x_1 + x_2 \qquad\qquad & \leq 5 \\
 & x_2 \qquad\qquad & \leq 1
\end{array}$$

$$-x_3 + 2x_4 \le 2$$
$$x_3 + 2x_4 \le 6$$
$$x_i \ge 0,\ i=1,2,3,4.$$

7. A company manufactures n different products, each of which uses various amounts of m limited resources. Each unit of product yields a profit of c_i dollars and uses a_{ji} units of j^{th} resource. The available amount of the j^{th} resource b_j. To maximize profit, the company selects the quantities $\{x_i\}$ to be manufactured of each product by solving:

$$\max_{\underline{x}} \quad \underline{c}^T \underline{x} \text{ subject to } A\underline{x} \le \underline{b},\ \underline{x} \ge 0$$

The unit profits $\{c_i\}$ already take into account the variable cost associated with manufacturing each unit. In addition to that cost, the company incurs a fixed overhead H, and for accounting purposes it wants to allocate this overhead to each of its products. In other words, it wants to adjust $\{c_i\}$ so as to account for the overhead. Such an overhead allocation scheme must meet two conditions: (1) since H is fixed regardless of the product mix, the overhead allocation scheme must not alter the optimal solution (i.e., product mix), and (2) all the overhead must be allocated. That is, the optimal value of the objective function with the modified cost coefficients must be H dollars lower than f^* - the original optimal value of the objective function.

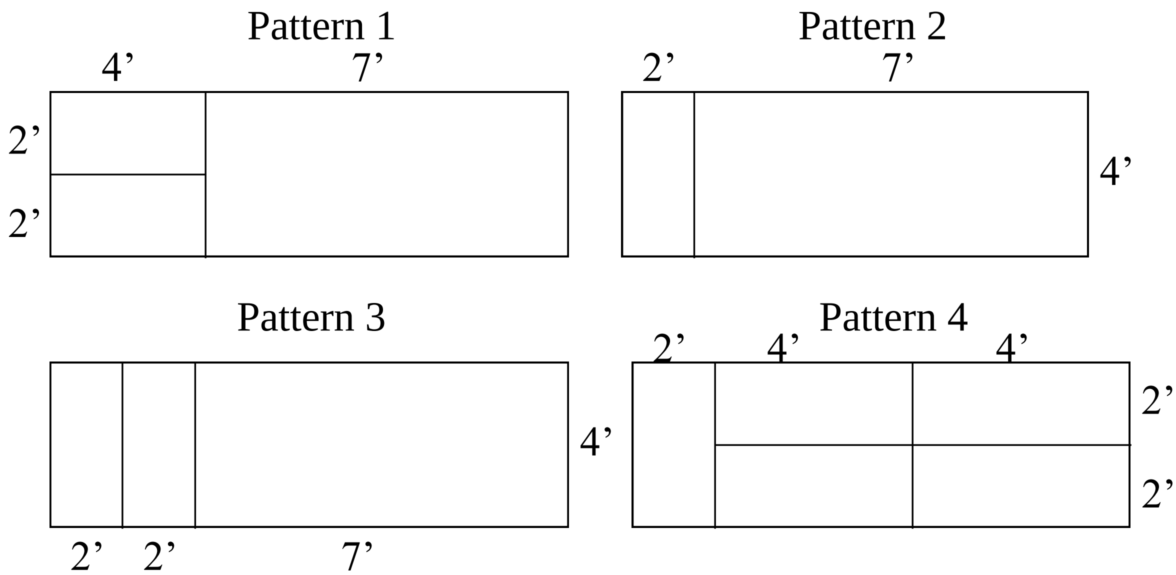
(a) Consider the allocation scheme in which the unit profits are modified according to $c_i' = c_i - r\,\boldsymbol{\lambda}^{*T}\boldsymbol{a}_i$, where $\boldsymbol{\lambda}^*$ is the optimal dual vector for the original objective, $\boldsymbol{a}_i$ is the i^{th} column of A, and $r = H/f^*$ (assume $H \le f^*$). Show that the optimal $\boldsymbol{x}$ for the modified problem is the same as that of the original problem and the new dual solution is $\boldsymbol{\lambda}^*_{new} = \boldsymbol{\lambda}^*(1-r)$. Show that the approach fully allocates H.

(b) Suppose that the overhead can be traced to each of the resources. Let $H_j \ge 0$ be the amount of overhead associated with the j^{th} resource, where

$$\sum_{j=1}^{m} H_j \le f^*, \text{ and } r_j = \frac{H_j}{b_j} \le \lambda_j^* \text{ for } j = 1,2,..,m$$

Based on this information, an allocation scheme has been proposed where the unit profits are modified such that $c_i' = c_i - \boldsymbol{r}^T\boldsymbol{a}_i$. Show that the optimal $\boldsymbol{x}$ for this modified problem is the same as that for the original problem, and that the corresponding dual solution is $\boldsymbol{\lambda}^*_{new} = \boldsymbol{\lambda}^* - \boldsymbol{r}$. Show that this scheme also fully allocates H_j.

8. A manufacturer of metal sheets received an order for producing 2000 sheets of size 2'x4' and 1000 sheets of size 4'x7'. Two standard sheets are available of sizes 10'x3000' and 11'x2000'. The engineering staff decided that the following four cutting patterns are suitable for this order. Formulate the problem of meeting the order and minimizing the waste as a linear program and solve it by the simplex method.



9. Exercise 3.22 of Text, page 134.

10. Exercise 6.3 of Text, page 261.