The University of Connecticut **Fall 2008**
Dept. of ECE **KRP**

Problem Set # 3

Due September 17, 2008

Part I: Computational (Due September 24, 2008)

Using MATLAB, compute $\int e^{A\sigma} BB^T \left(e^{A\sigma}\right)^T d\sigma$, investigate how you can stabilize an unstable system.

1. Consider both SISO and MIMO systems of order up to 6.
2. Consider completely controllable **and** stabilizable systems.
3. Consider systems with clustered and widely separated eigen values. Especially, consider systems in standard controllable (or phase variable) form.

Part II: Analytical (Due September 17, 2008): Do only Problems 2 and 4

1. Show that if a system $d\underline{x}/dt = A\underline{x} + B\underline{u}$ is completely controllable, then $\underline{u} = -L\underline{x}$ with $L = B^T W^{-1}(t_f, \beta)$, where
$$W\left(t_f, \beta\right) = \int_0^{t_f} e^{-(A+\beta I)\sigma} BB^T \left[e^{-(A+\beta I)\sigma}\right]^T d\sigma; \quad t_f = \text{arbitrary}$$
results in a closed-loop system matrix $A = (A - BL)$ with $\mathrm{Re}\left[\lambda_i(A)\right] < -\beta$ $(\beta \geq 0)$.

2. Suppose one has accurately computed $\Gamma = \int_0^{t_f} e^{A\sigma} B d\sigma$ and now wishes to compute $S = \int_0^{t_f} e^{A\sigma} BB^T e^{A^T\sigma} d\sigma$. Find a correlation scalar c_1 such that $\Gamma\Gamma^T + c_1 A\Gamma\Gamma^T A^T$ is an approximation to S that is valid through to terms in T^4. What is the error in the T^5 terms?
3. Show that $S = \int_0^{t_f} e^{A\sigma} \underline{b}\underline{b}^T e^{A^T\sigma} d\sigma$, where $\underline{b}$ is an n-column vector, can be written as an infinite summation of the form:
$$S = \left[\underline{b} \quad A\underline{b} \quad A^2\underline{b} \quad \cdots\right] = \begin{bmatrix} \frac{1}{1!} & \frac{1}{2!} & \frac{1}{3!} & \cdots \\ \frac{1}{2!} & \frac{2}{3!} & \frac{3}{4!} & \cdots \\ \frac{1}{3!} & \frac{3}{4!} & \ddots & \cdots \\ \cdots & \cdots & \cdots & \ddots \end{bmatrix} \begin{bmatrix} \underline{b} \\ [A\underline{b}]^T \\ \vdots \\ \vdots \end{bmatrix}$$

4. Consider the following 3 by 3 block bi-diagonal matrix that arises in the modeling of fault-tolerant computer systems.

$$R = \begin{bmatrix} 0 & 0 & 0 \\ F & A & 0 \\ 0 & F & A \end{bmatrix}$$

(a) Show that e^{Rt} has a lower triangular block structure with only 4 distinct blocks and is of the form:

$$e^{Rt} = \begin{bmatrix} I & 0 & 0 \\ \Gamma_1 & \Phi_1 & 0 \\ \Gamma_2 & \Phi_2 & \Phi_1 \end{bmatrix}$$

(b) Show that Γ_i and Φ_i can be evaluated recursively via the doubling formulas for i = 2, 1:

$$\Gamma_i(2t) = \Gamma_i(t) + \sum_{j=1}^{i} \Gamma_j(t)\Phi_{i-j+1}(t)$$

$$\Phi_i(2t) = \sum_{j=1}^{i} \Phi_j(t)\Phi_{i-j+1}(t)$$

(c) (Bonus) How would you evaluate Γ_i and Φ_i via the pade approximation and how would you generalize for n x n block bi-diagonal structures? See my 1990 paper in IEEE Trans. On Computers.