The University of Connecticut **Fall 2012**
Dept. of ECE **KRP**

ECE 6095
Problem Set # 4
(Due October 16, 2012)

1. For the feed-forward (also known as two-degree of freedom) control structure of lecture 2, establish the matrix of relationships among the independent and dependent variables shown in the lecture notes.

2. Consider digital state variable feedback for the continuous system

$$\dot{\underline{x}}(t) = A\underline{x}(t) + Bu(t), \quad y(t) = C\underline{x}(t)$$

with $A = \begin{bmatrix} -0.6 & 1 \\ -0.2 & -0.4 \end{bmatrix}$ $\quad B = \begin{bmatrix} 3.5 \\ 1.5 \end{bmatrix}$ $\quad C = \begin{bmatrix} 2 & -3.5 \end{bmatrix}$

(a) Suggest a suitable range of time step h for use in digital control.
Fix $h = 0.2$sec. Using the algorithm $u(k) = K_r r(k) - K\underline{x}(k)$ with $K = \begin{bmatrix} 2.5 & -5 \end{bmatrix}$, and $K_r = 2$, answer the following questions:
(b) What are the poles/eigenvalues of the closed-loop system?
(c) What are ξ and ω_n for the dominant pair of second-order poles?
(d) If the system is stable, what is its phase margin, ϕ_m?
(e) What is the closed-loop transfer function, T(z) between r(k) and y(k)?
(f) If K = [2.5 $-\alpha$], sketch the root-locus of the closed-loop system poles as a function of α for $0 < \alpha < \infty$.

3. In problem 2 select K_r so that $K_p = \infty$. With this choice of K_p, what is K_v?
4. Establish the following relation for K_v in terms of closed-loop poles and zeros:

$$\frac{1}{K_v} = h\left[\sum \frac{1}{1-p_i} - \sum \frac{1}{1-z_i}\right]$$

5. Series compensation u(z) = H(z)e(z) will now be considered for Problem 2:
(a) Still using h = 0.2, what is $\tilde{G}(z)$ for the open-loop system?
(b) If $H(z) = \beta_o = 0.5$, is the system stable? What are K_p and K_v? Sketch the root-locus of the CL poles for $\beta_o > 0$, and show the poles for β_o=0.5.
(c) Since $K_p \neq \infty$ in the above, the addition of a small integral term to H(z) is suggested, viz.,

$$H(z) = 0.5 + \frac{0.1}{1-z^{-1}} = \frac{0.6z - 0.5}{z-1}$$

What are K_p and K_v now?

(d) What is the phase margin, ϕ_m, for each of the above two choices of H(z) in (b) and (c)? Explain the differences and discuss. If in case (c), a time delay τ is introduced into the feedback loop, how large can τ be before the system becomes unstable, i.e., what is $\tau_{\max}$?

(e) If $r(t) = 2 \sin 1.2t$, what's the magnitude of the s.s. error, |e(k)|, in case (c)?

(f) Simulate the time response for both of the cases (b) and (c) over the time interval [0, 15] when $\underline{x}(0) = 0$, and r(t) is a unit step. Use NS = 2. From the curves what is the settling time t_s and % overshoot. Do these numbers agree with what you expect on the basis of the closed-loop pole locations?

6. Repeat the simulation for case (c) above, but introduce an uncompensated time-delay with value $\tau_{\max}$ in the system. Is the system just on the verge of instability? Can you find the values of the poles of the discrete closed-loop system for this case, i.e., the poles of $1 + \widetilde{G}(z)H(z)$?

7. Consider a first order system $G(s) = 1/(10s+1)$ and a series compensator $H(s)=k$. Assume unity feedback.
   a. Discretize the system using $h=1$ second
   b. Find the least positive gain k so that the following are true:
      i. Magnitude of steady-state error ≤ 0.1 for unit step input when $v=d=0$.
      ii. Magnitude of output ≤ 0.1 for all $d(t)$ such that $\|d\|_2 \leq 1$ when $r=v=0$.

8. Show that any SVFB system $\underline{x}(k+1) = \Phi \underline{x}(k) + \Gamma u(k)$, $y(k) = C\underline{x}(k)$, $u(k) = K_r r(k) - K\underline{x}(k)$ can be viewed as an equivalent feedback compensation design with

$$u(z) = K_r r(z) - H_{eq}(z) y(z)$$

such that "both designs" have the same closed-loop transfer function. What is $H_{eq}(z)$ in terms of Φ, Γ, K, C ? In the above problem 2(a) what is $H_{eq}(z)$?