The University of Connecticut **Fall 2008**
Dept. of ECE **KRP**

Problem Set # 4
(Due September 24, 2008)

LU Decomposition

Computational (Due October 1, 2008)
Using MATLAB's LU decomposition routine, solve $A\underline{x} = \underline{b}$ and

1. Employ an iterative improvement scheme to refine the solution
2. Compute an estimate of the condition number $\kappa(A)$

Extensively test the program, report and discuss your results and conclusions. Does balancing A before solving $A\underline{x} = \underline{b}$ have any significant impact?

Analytical (Do problems 1,2, 5, 6 and 8)

1. Describe the vectors in the column space, null space, row space, and left null space of

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Find the *LU* factorization of:

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

   Do you see any relationship between L and U ? Solve $A\underline{x} = \underline{b}$ if $\underline{b}$ has components [1, 0, 0, 0]

3. Show that if an n by n matrix A has an LU decomposition and is nonsingular, then L and U are unique.

4. Discuss how the determinant of a square nonsingular matrix can be obtained with minimum risk of overflow and underflow

5. Suppose $\underline{x} = A^{-1}\underline{b}$. If $\underline{e} = \underline{x}^* - \underline{x}$ (the error) and $\underline{r} = \underline{b} - A\,\underline{x}^*$ (the residual), then

$$\|\underline{r}\|\left[\|A\|\right]^{-1} \le \|\underline{e}\| \le \|A^{-1}\|\ \|\underline{r}\|$$

6. Consider the linear system $A\underline{x} = \underline{b}$ with

$$A = \begin{bmatrix} 7 & 9 & 2 \\ 6 & 4 & 1 \\ 3 & 9 & 3 \end{bmatrix}; \quad \underline{b} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$

a) Solve this linear system using LU factors (use MATLAB)

b) Suppose that the right hand side is changed to :

$$\underline{b} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$

Solve the modified system without refactoring the matrix A

c) Suppose that it was discovered that a mistake had been made when collecting data for the matrix A, and that the (3, 3) entry should have been $a_{33} = 4$. Using the Sherman-Morrison-Woodbury formula, determine the solution of the modified system without refactoring the matrix A.

7. A paint company is trying to use up excess quantities of four shades of green paint by mixing them to form a more popular shade. One gallon of the new paint will be made of x_1 gallons of paint 1, x_2 gallons of paint 2, etc. Each of the paints is made up of four pigments, and they are related by the following system of linear equations:

$$\begin{bmatrix} 80 & 0 & 30 & 10 \\ 0 & 80 & 10 & 10 \\ 16 & 20 & 60 & 72 \\ 4 & 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 40 \\ 27 \\ 31 \\ 2 \end{bmatrix}$$

Each number represents a percentage; for example, paint 3, contains 10% of pigment 2, and the more popular shade should contain 40% of pigment 1. Solve this system using MATLAB. What is the condition number of the matrix?

8. Consider the linear system of equations $A\underline{x} = \underline{b}$ with

$$\begin{bmatrix} 21 & 67 & 88 & 73 \\ 76 & 63 & 7 & 20 \\ 0 & 85 & 56 & 54 \\ 19.3 & 43 & 30.2 & 29.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 141 \\ 109 \\ 218 \\ 93.7 \end{bmatrix}$$

a) Solve the linear system in single precision. You may use the routine you have written.
b) Perform iterative improvement on the solution.
c) What is the condition number of *A?*