

Homework Set # 5
(Due: March 10, 2016)

1. Solve the following problem by the dual simplex method:

$$\min -7x_1 + 7x_2 - 2x_3 - x_4 - 6x_5$$

$$s.t. 3x_1 - x_2 + x_3 - 2x_4 = -3$$

$$2x_1 + x_2 + x_4 + x_5 = 4$$

$$-x_1 + 3x_2 - 3x_4 + x_6 = 12$$

$$x_i \geq 0, i = 1, 2, \dots, 6$$

2. A textile firm is capable of producing three products - x_1, x_2, x_3 . Its production plan for next month must satisfy the constraints:

$$x_1 + 2x_2 + 2x_3 \leq 12$$

$$2x_1 + 4x_2 + x_3 \leq f$$

$$x_i \geq 0, i = 1, 2, 3$$

The first constraint is determined by equipment availability and is fixed. The second constraint is determined by the availability of cotton. The net profits of the products are 2, 3, and 3, respectively, exclusive of the cost of cotton and fixed costs. Find the shadow price λ_2 of the cotton input as a function of f . (Hint: Use the dual simplex method). Plot $\lambda_2(f)$ and the net profit $z(f)$ exclusive of the cost for cotton.

3. a) Use the revised simplex method and solve:

$$\min 2x_1 + 3x_2 + 2x_3 + 2x_4$$

$$s.t. x_1 + 2x_2 + x_3 + 2x_4 = 3$$

$$x_1 + x_2 + 2x_3 + 4x_4 = 5$$

$$x_i \geq 0, i = 1, 2, 3, 4$$

b) Using the work done in Part (a) and the dual simplex method, solve the same problem but with the right-hand sides of the equations changed to 8 and 7, respectively.

4. Consider the problem:

$$\min 2x_1 + x_2 + 4x_3$$

$$s.t. x_1 + x_2 + 2x_3 = 3$$

$$2x_1 + x_2 + 3x_3 = 5$$

$$x_i \geq 0, i = 1, 2, 3$$

a) What is the dual problem?

b) Note that $\lambda = (1, 0)$ is feasible for the dual. Starting with this λ , solve the primal using the primal-dual algorithm.

5. Exercise 9.11 of Text (Hint: convert to two variable problem via $x_3 = 1 - x_1 - x_2$.)