University of Connecticut **Fall 2008**
ECE Dept. **KRP**

ECE 357
<u>Problem Set # 5</u>
Due October 1, 2008

<u>Part A: Analytical</u> (Do Problems 1,2,3,4, 8, 9, 15 and 16)

1. The Cholesky decomposition discussed in class computed a column at a time. Formulate a Cholesky decomposition algorithm that computes a row at a time (row version).
2. Suppose $A \in R^{n\times n}$ is symmetric and positive definite. Give an algorithm for computing an upper triangular matrix $R \in R^{n\times n}$ such that $A=RR^T$.
3. Show that the Cholesky decomposition $A=SS^T$ is unique for positive definite symmetric matrices.
4. Suppose $A=I+\underline{u}\underline{u}^T$ where $A\in R^{n\times n}$ and $\|\underline{u}\|_2=1$. Give explicit formulate for the diagonal and subdiagonal's of A's Choleskt factor.
5. Find the LDL^{T} and SS^T factorizations for the matrix:

$$A=\begin{bmatrix}1 & 2 & 4\\ 2 & 7 & 14\\ 4 & 14 & 29\end{bmatrix}$$

   Solve $A\underline{x}=\underline{b}$ when $b=[0\ 6\ 1]^T$ using the two triangular systems computed. How do you know that A is positive definite?
6. For any $\underline{y}\in R^n$, define the vectors $\underline{y}_+=½[\underline{y}+E_n\underline{y}]$ and $\underline{y}_-=½[\underline{y}-E_n\underline{y}]$. For an $n\times n$ symmetric Toeplitz matrix T_n, show that $T_n\underline{x}=\underline{b}$ can be written as:

$$T_n\underline{x}_+=\underline{b}_+ \text{ and } T_n\underline{x}_-=\underline{b}_-.$$

7. If a new row $\underline{v}^T$ is added to A, what is the change in A^TA?
8. Under what conditions is a Householder matrix $[I-2\underline{v}\,\underline{v}^T/\underline{v}^T\underline{v}]$ persymmetric?
9. Suppose $\underline{x} \in R^n$ and that $Q \in R^{n\times n}$ is orthogonal. Show that if

$$S=[\underline{x}, Q\underline{x}, \ldots, Q^{n-1}\underline{x}]$$

   Then S^TS is Toeplitz.
10. Explain (and if possible code) the following solution of $A\underline{x}=\underline{b}$ for positive definite tridiagonal A. The diagonal of A is originally in $d_1, d_2, \ldots, d_n$ and the subdiagonal and super diagonal in $l_1, l_2, \ldots, l_{n-1}$; the solution $\underline{x}$ overwrites $\underline{b}$.

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For k=2, …, n
        t=l_{k-1}
        l_{k-1}=t/d_{k-1}
        d_k=d_k-t l_{k-1}
EndDO
For k=2, …, n
        b_k=b_k-l_{k-1} b_{k-1}
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EndDO

For k=1, 2, …, n

$b_k = b_k / d_k$

EndDO

For k=n-1, …, 1

$b_k = b_k - l_k b_{k+1}$

EndDO

show how this uses $5n$ multiplications and divisions, and give an example of failure when A is not positive definite.

11. Show the following equivalence:

$$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + A^{-1}BSB^TA^{-1} & -A^{-1}BS \\ -SB^TA^{-1} & S \end{bmatrix}$$

where $S=(C-B^TA^{-1}B)^{-1}$, A and C are square, but B can be rectangular.

12. For the block quadratic form

$$f = \begin{bmatrix} \underline{x}^T & \underline{y}^T \end{bmatrix} \begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{y} \end{bmatrix}$$

Find the term that completes the squares:

$$f=(\underline{x}+A^{-1}B\underline{y})^TA(\underline{x}+A^{-1}B\underline{y})+\underline{y}^T(?)\underline{y}.$$

The block matrix is positive definite when A and $(C-B^TA^{-1}B)$ are positive definite

13. If each diagonal entry a_{ii} is larger than the sum of the absolute values $|a_{ij}|$ along the rest of its row, then the symmetric matrix A is positive definite. How large would c have to be in an $n\times n$ matrix $A=(c-1)I+\underline{e}\underline{e}^T$, where $\underline{e}$ is a vector of 1s for this statement to apply? How large does c actually have to be to assure that A is positive definite?

14. The inverse of $B=(I-\underline{v}\underline{v}^T)$ has the form $B^{-1}=(I-c\underline{v}\underline{v}^T)$. By multiplication, find the value of c. Under what condition on $\underline{v}$ is B not invertible?

15. Prove that if $AA^T\underline{y}=0$ then $A^T\underline{y}=0$. The matrices AA^T and A^T have the same null space, the same row space, and the same rank. Find all three if:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ -1 & 0 \end{bmatrix}$$

16. Given n-dimensional vectors $\underline{x}$ and $\underline{y}$, what combination $(\underline{x}+a\underline{y})$ is A-orthogonal to $\underline{x}$?

17. Using Caley-Hamilton theorem, show that $\underline{x}=A^{-1}\underline{b}$ is a linear combination of ($\underline{b}$, $A\underline{b}$, …, $A_{n-1}\,\underline{b}$). If A has only two non-zero Eigen values λ_1, λ_2, what combination $cI+dA$ is equal to A^{-1}?

<u>Part B: Computational</u> (Due October 22, 2008)

Extensively test MATLAB's Cholesky decomposition routine using the various test matrices.