The University of Connecticut **Fall 2012**
Dept. of ECE **KRP**

ECE 6095/4121
<u>Problem Set # 6</u>
(Due November 13, 2012)

1. Implement and evaluate Tustin equivalents of the analog controllers you designed for problems 1 through 6 of HW set # 5 by comparing the responses of the digital controllers to those corresponding to the analog controllers.
2. For what range of K are the systems with the following characteristic polynomials stable?

   (i) $P(z) = z^3 + 1.2z^2 + Kz - 0.2$

   (ii) $P(z) = z^2 - (K + 0.1)z + 2.3K - 0.6$

   Reconcile your results in (i) and (ii) with a root-locus sketch.

3. Consider the position control of the satellite system with transfer function G(s) = y(s)/u(s) = $1/s^2$. Use digital control with a time step h = 1. You will design and implement a PD controller in "modified derivative on output" form:

$$u(z) = K\left[e(z) - \beta \frac{2}{h}\frac{z-1}{z+1} y(z)\right] \qquad e(z) = r(z) - y(z)$$

   (Note that this controller is obtained from the continuous feedback control law u(t) = $K[e(t) - \beta\, \dot{y}(t)]$ using Tustin equivalence)

   (a) What is the closed-loop transfer function from r to y? What are the conditions on K and β for closed-loop stability?
   (b) Let β = 0 and sketch the root locus of the closed-loop system poles. Are there any values of K for which the CL system is stable? Repeat for β = 3/2.
   (c) Let β = 3/2. Assume the satellite is initially pointing at an angle of 1.5rad, and we wish to suddenly command it to move to an angle of 2.5rad. Examine the closed-loop response of y(t), y(kh), and u(kh) over [0, 10]. Do this for K = 0.5, 1 and 1.25. Let NS = 4 in your simulation. Reconcile your results with the corresponding pole locations on the root-locus. DISCUSS.

4.
   a. Compare the Bode plot of the lag network $H(s) = 9.32\frac{1.1s+1}{6.3s+1}$ (which came from a G(s)H(s) with $\omega_c \approx 1.9$) with that of:
      (a) the Backward Difference equivalent
      (b) the Tustin equivalent
      (c) the Tustin equivalent with a prewarp $\omega_1 = 1.9$
      (d) the Pole-zero mapping equivalent

(e) A zero-order hold equivalent (i.e., like what you use to get $\tilde{G}(z)$)
First use a time step h = 0.4sec, and then try h = 1.2sec. Any comments?

b. Is a Tustin equivalent approach $H(s) \rightarrow \tilde{H}(z)$ likely to work more often for a system with a lag compensator than for one with a lead compensator? Why?

c. Since a Tustin approach tends to give an $\tilde{H}(z)$ that closely approximates a given H(s) over $\omega \in \left[0, \frac{1}{h}\right]$ why is it that we don't use a Tustin transformation to obtain $\tilde{G}(z)$ from G(s) for doing our discrete design? EXPLAIN.

5. "There you are-our new control engineer!" exclaims the boss as she bursts into your office. She hands you a sheet of paper upon which is written what you instantly recognize as a continuous time, second-order system in state space form:

$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t); \qquad y(t) = x_1(t)$$

"We need to do some digital state variable feedback control for this servomotor system they are going to use out on the shop floor," she says. "x_1 is the position of the motor/lathe shaft…and we'll use a sample time of 0.1 sec".

(a) Find an equation for the gain vector $\underline{K} = [K_1, K_2]$ to achieve digital closed-loop pole locations λ_1 and λ_2. [Recall $u(k) = K_r r(k) - K_1 x_1(k) - K_2 x_2(k)$]

(b) The shop foreman first wanted a controller that would reduce any initial offset in y to zero. So, the guys on the floor went and implemented a simple unity feedback $u(k) = -0.2y(k)$. The foreman felt that the corresponding over-shoot was OK, but he wanted a settling time-constant $\tau \sim 1\text{sec}$. Using the equations from part (a), find the appropriate gain vector to achieve his specs. (You may wish to check your result using Ackermann's formula or place or acker command.)

(c) On Tuesday, the shop foreman changes his mind, he now wants a controller that will return <u>any initial state</u> $\underline{x}(0)$ to $\underline{0}$ in 0.2 sec. While at lunch, you see a guy who has owed you $30 since high school days. Suddenly, you remember how to give the foreman what he wants. Verify that your design holds for three different initial conditions. Show closed-loop responses. What are ω_c, ϕ_m?

(d) On Thursday the shop foreman decides that he now wants to be able to input a position command r (so that y will go to r in steady-state). Modify the design in (c) accordingly. Provide a step response for r = 3.5 and $\underline{x}(0) = \underline{0}$.