Lecture 2: Feedback Structures, Digital Control and Stability

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ECE 6095/4121
Digital Control of Mechatronic Systems

Mechatronic Systems Introduction & Overview

1. Feedback Control Structures for Continuous-time Systems
   - Output feedback (series compensation, Proportional-Integral-Derivative (PID), Feed forward-feedback, Internal Model Control…)
   - State variable feedback

2. Classification of Control Design Techniques

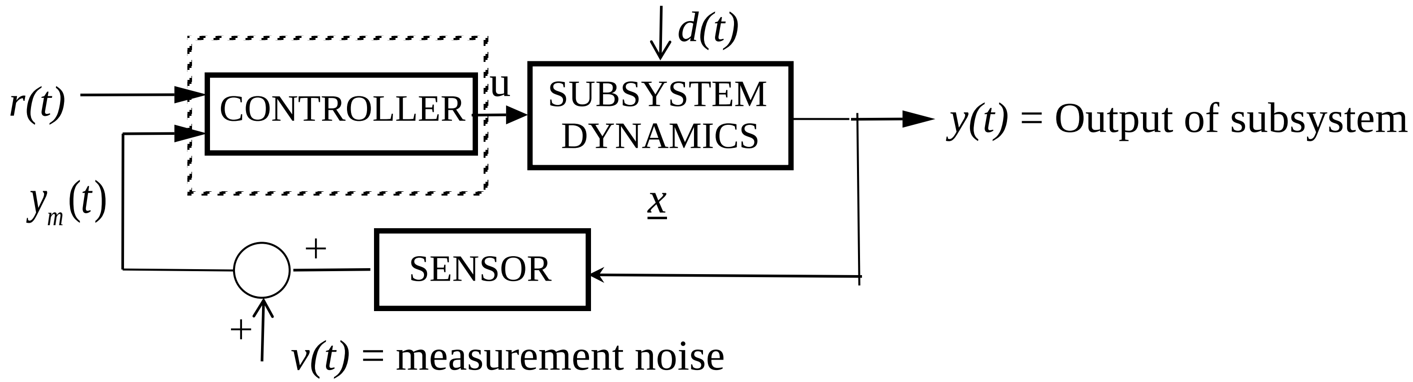
3. Digital Control Loop Structure
   - Relationships among time signals
   - Typical algorithm implementation considerations

4. Discrete-time System Stability

5. Continuous-time-vs.-Discrete-time Relationships
   - $s \rightarrow z$ plane mapping

Basic Feedback Control Loop – SISO System-1

- Consider a particular subsystem under local computer control.



The feedback control problem is to design the controller to have $y(t) \approx r(t)$ = reference setpoint, even in the face of external disturbances, imprecise models of dynamics, etc.

Error, $e(t) \triangleq r(t) - y(t)$.

- We define $u(t)$ = control signal produced by the controller. This is the input signal to the subsystem (e.g., valve opening, fuel flow, etc.)
  If only measurements of the system output are available:

$$\textit{unknown } d(t): u(t) = H\left[r(\bullet), y_m(\bullet)\right]; \textit{ measured } d(t): u(t) = H\left[r(\bullet), d(\bullet), y_m(\bullet)\right]$$

Output Feedback

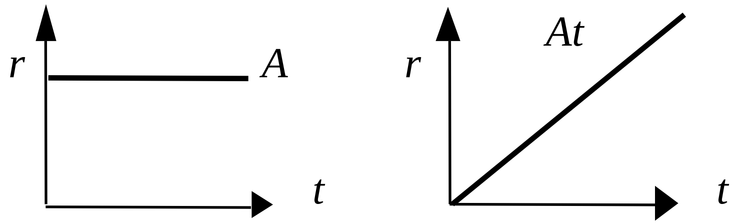
Basic Feedback Control Loop – SISO System-2

- Often we have measurements of the system state $\underline{x}$ available for feedback, in which case:

$$u(t) = H\left[r(\bullet), \underline{x}(\bullet)\right]$$ <u>State Variable Feedback</u>

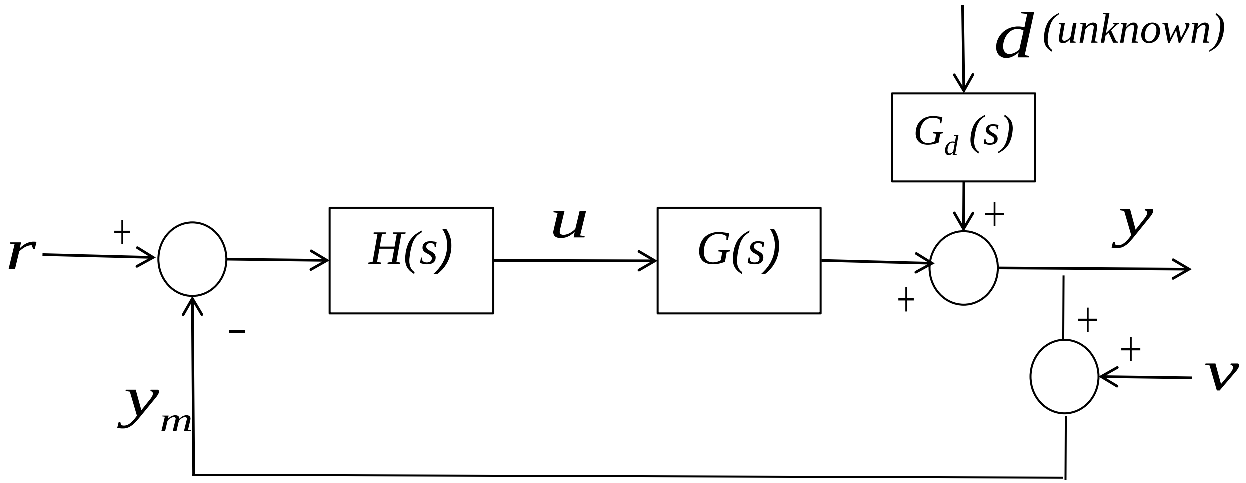
- When $r(t) = 0$, we desire $y \simeq 0$. This is a **regulator** problem, where we wish to bring the system to the rest state (e.g., reduce the spin of a satellite, Linearized non-linear system around an equilibrium point, maintain speed (slip) variations near zero in an induction motor).

- When $r(t) \neq 0$, we have an **output command (servo)** problem, where we wish to reduce $e(t) \to 0$. Typically, $r(t)$ is a step or ramp command.



(Key Relation in Output Feedback: $T(s) + S(s) = 1$)-1

- Single degree of freedom controller $u(t) = H\left[r(t), y_m(t)\right]$



$$y(s) = G(s)u(s) + G_d(s)d(s) = G(s)H(s)[r(s) - y(s) - v(s)] + G_d(s)d(s)$$

$$\Rightarrow y(s) = \underbrace{\frac{G(s)H(s)}{1+G(s)H(s)}}_{T(s)}[r(s) - v(s)] + \underbrace{\frac{G_d(s)}{1+G(s)H(s)}}_{T_d(s)}d(s)$$

(Key Relation in Output Feedback: $T(s) + S(s) = 1$)-2

$$y(s) = G(s)u + G_d(s)d(s) = T(s)[r(s) - v(s)] + T_d(s)d(s)$$

$$Loop\ Gain = G(s)H(s)$$

$$Return\ Difference = 1 + Loop\ Gain = 1 + G(s)H(s)$$

$$S(s) = \frac{1}{1+G(s)H(s)} = \frac{dT(s)/T(s)}{dG(s)/G(s)} = \frac{G(s)}{T(s)}\frac{dT(s)}{dG(s)} = S_G^T = Bode\ Sensitivity\ function$$

$$\frac{\Delta T(s)}{T(s)} = S(s)\frac{\Delta G(s)}{G(s)}$$

$$T(s) = \frac{G(s)H(s)}{1+G(s)H(s)} = \frac{y(s)}{r(s)}\Big|_{d=0,v=0} = G(s)H(s)S(s) = 1 - S(s) = Complementary\ Sensitivity\ function$$

$$T_d(s) = \frac{G_d(s)}{1+G(s)H(s)} = \frac{y(s)}{d(s)}\Big|_{r=0,v=0} = G_d(s)S(s)$$

Fundamental Tradeoff in Control Design: $T(s) + S(s) = 1$

$\Rightarrow$ *We cannot reduce (increase) both* $T(s)$ *and* $S(s)$ *simultaneously!*

Frequency Response:

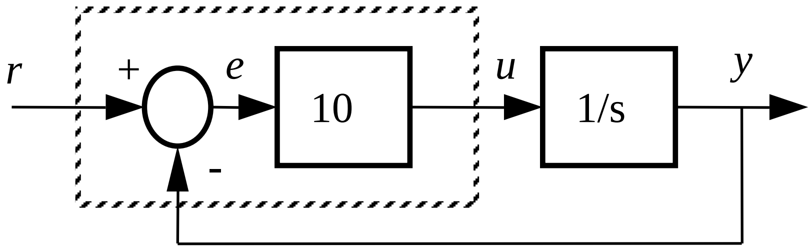
$$T(j\omega) + S(j\omega) = 1$$

$$\Rightarrow \mathrm{Re}[T(jw)] + \mathrm{Re}[S(j\omega)] = 1$$

$$\mathrm{Im}[T(j\omega)] + \mathrm{Im}[S(j\omega)] = 0$$

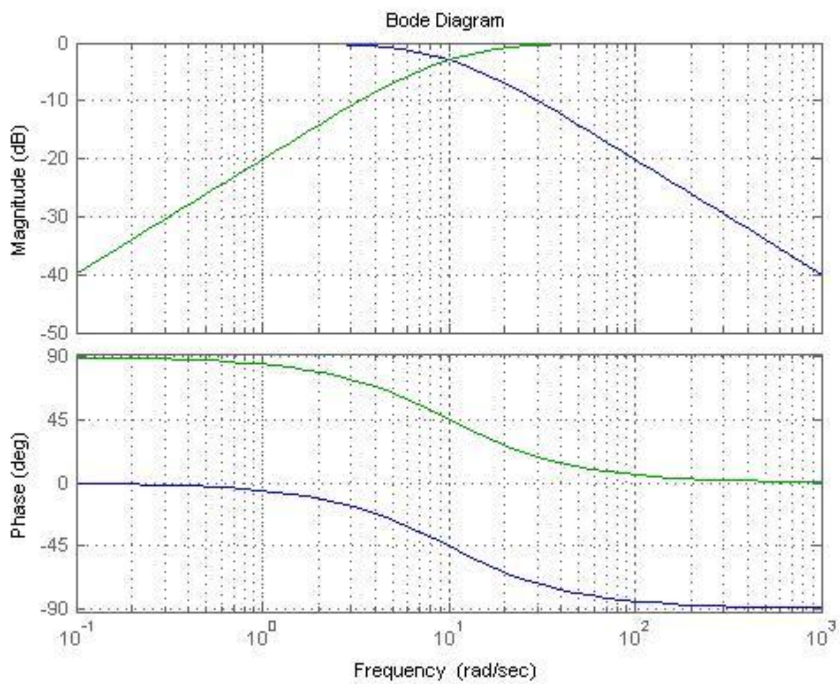
Let us look at output signal $y(s)$, *control signal* $u(s)$, *and error signal* $e(s)$

Illustration of $T(s) + S(s) = 1$



Closed-loop Transfer Function $T(s) = \frac{10}{s+10} \Rightarrow LPF$

Sensitivity Function $S(s) = \frac{s}{s+10} \Rightarrow HPF$



Note: in **linear** scale

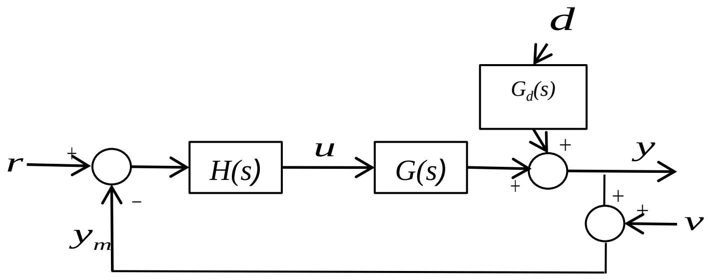
- at ω = 0.1 rad/sec.
  T(0.1j) = 0.9999-0.01j
  S(0.1j) = 0.0001+0.01j
- at ω = 10 rad/sec.
  T(10j) = 0.5-0.5j
  S(10j) = 0.5+ 0.5j
- at ω = 20 rad/sec.
  T(20j) = 0.2-0.4j
  S(10j) = 0.8+ 0.4j

Fundamental Tradeoffs in SISO Control Design

- Signals *y(s)*, *u(s)* and *e(s)* = *r(s)*-*y(s)*

Independent variables: r, d, v

Dependent variables: $y, e = r - y, u$



$$y(s) = T(s)[r(s) - v(s)] + S(s)G_d(s)d(s)$$

$$e(s) = r(s) - y(s) = S(s)r(s) - S(s)G_d(s)d(s) + T(s)v(s)$$

$$u(s) = H(s)[r(s) - y_m(s)] = H(s)[r(s) - y(s) - v(s)] = H(s)S(s)[r(s) - v(s) - G_d(s)d(s)]$$

$$\begin{bmatrix} y(s) \\ e(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} T(s) & S(s)G_d(s) & -T(s) \\ S(s) & -S(s)G_d(s) & T(s) \\ H(s)S(s) & -H(s)S(s)G_d(s) & -H(s)S(s) \end{bmatrix} \begin{bmatrix} r(s) \\ d(s) \\ v(s) \end{bmatrix}$$

Typical design spec.

$$|S(j\omega)| \le M_s(\omega)\ \forall \omega$$

and $|T(j\omega)| \le M_T(\omega)\ \forall \omega$

$$\min_{H(s)} [|W_1(j\omega)S(j\omega)|^2 + |W_2(j\omega)T(j\omega)|^2 + |W_3(j\omega)H(j\omega)S(j\omega)|^2]^{1/2}$$

Measurement noise v is typically a high-frequency signal, while r and d are low-frequency signals

Design Rationale: Keep $S(s)$ small at low frequencies and $T(s)$ small at high frequencies

Low frequencies: Large $G(s)H(s) \Rightarrow$ Small $S(s)$ and large $T(s) \approx 1$ (Recall: $S(s) + T(s) = 1$)

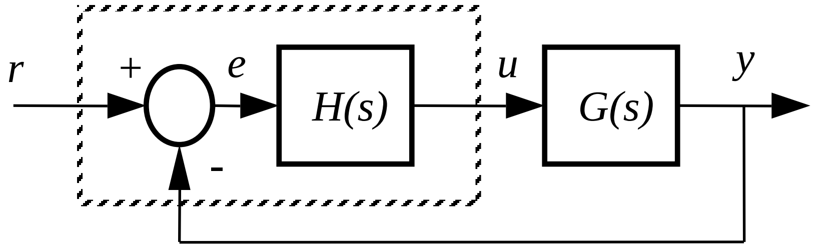
$\Rightarrow$ Good command tracking and disturbance rejection. Fortunately, v is negligible at low frequencies

High frequencies: Small $G(s)H(s) \Rightarrow$ Small $T(s)$ and large $S(s) \approx 1$

$\Rightarrow$ Attenuate measurement noise. Fortunately, both command and disturbance signals are negligible.

Also, need to make sure that the control signal does not saturate!

Output Feedback-1



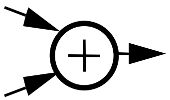
$$T(s) = \frac{G(s)H(s)}{1 + G(s)H(s)} = \frac{y(s)}{r(s)}$$

$$u(s) = H(s)e(s) = H(s)\left[r(s) - y(s)\right]$$

- Design objective is to determine the transfer function *H(s)* so that *y(t)* → *r(t)* "nicely" and the closed-loop has desirable stability/transient response. Usually, *H(s)* has a simple form, e.g.,

$$H(s) = K\frac{1 + s/\alpha\omega_1}{1 + s/\omega_1} \qquad \alpha > 1$$ LAG COMPENSATOR

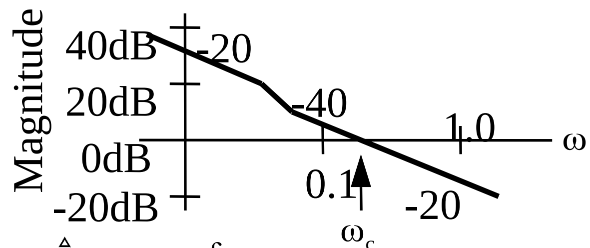
$$H(s) = K\frac{1 + s/\omega_2}{1 + s/\beta\omega_2} \qquad \beta > 1$$ LEAD COMPENSATOR



$$H(s) = K_0 + \frac{K_1}{s} + \frac{K_2 s}{\tau_2 s + 1}$$ PID COMPENSATOR (~LAG-LEAD)

Output Feedback-2

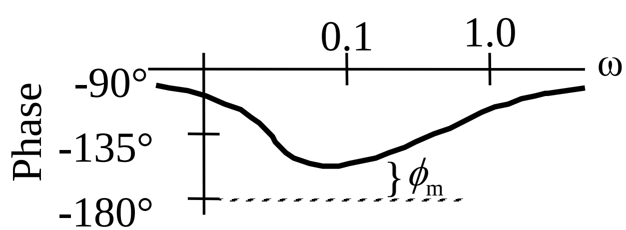
- Selection of design parameters ($K, \alpha, \omega_1, \ldots$) via either root locus or Bode plot methods => "classical design", using properties of loop gain, ***GH***.



$\omega_c \triangleq$ crossover frequency

place at which $|GH|_{j\omega_c} = 1$

ω_c is a measure of CL system bandwidth



$\phi_m \triangleq$ phase margin $= \pi + \measuredangle GH|_{\omega_c}$

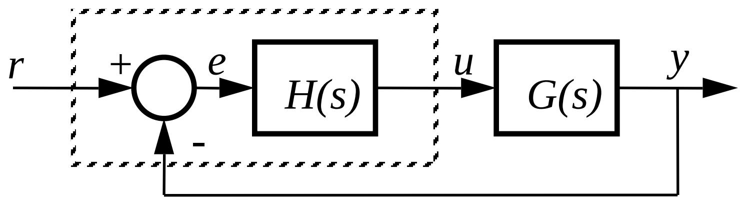
ϕ_m is a measure of CL stability and tolerance to loop delays

$$\left[\begin{array}{l} \text{Recall } \measuredangle e^{-s\tau}\Big|_{s=j\omega_c} = -\omega_c \tau \\ \Rightarrow \tau_{max} = \phi_m / \omega_c \end{array}\right]$$

A good design will have:

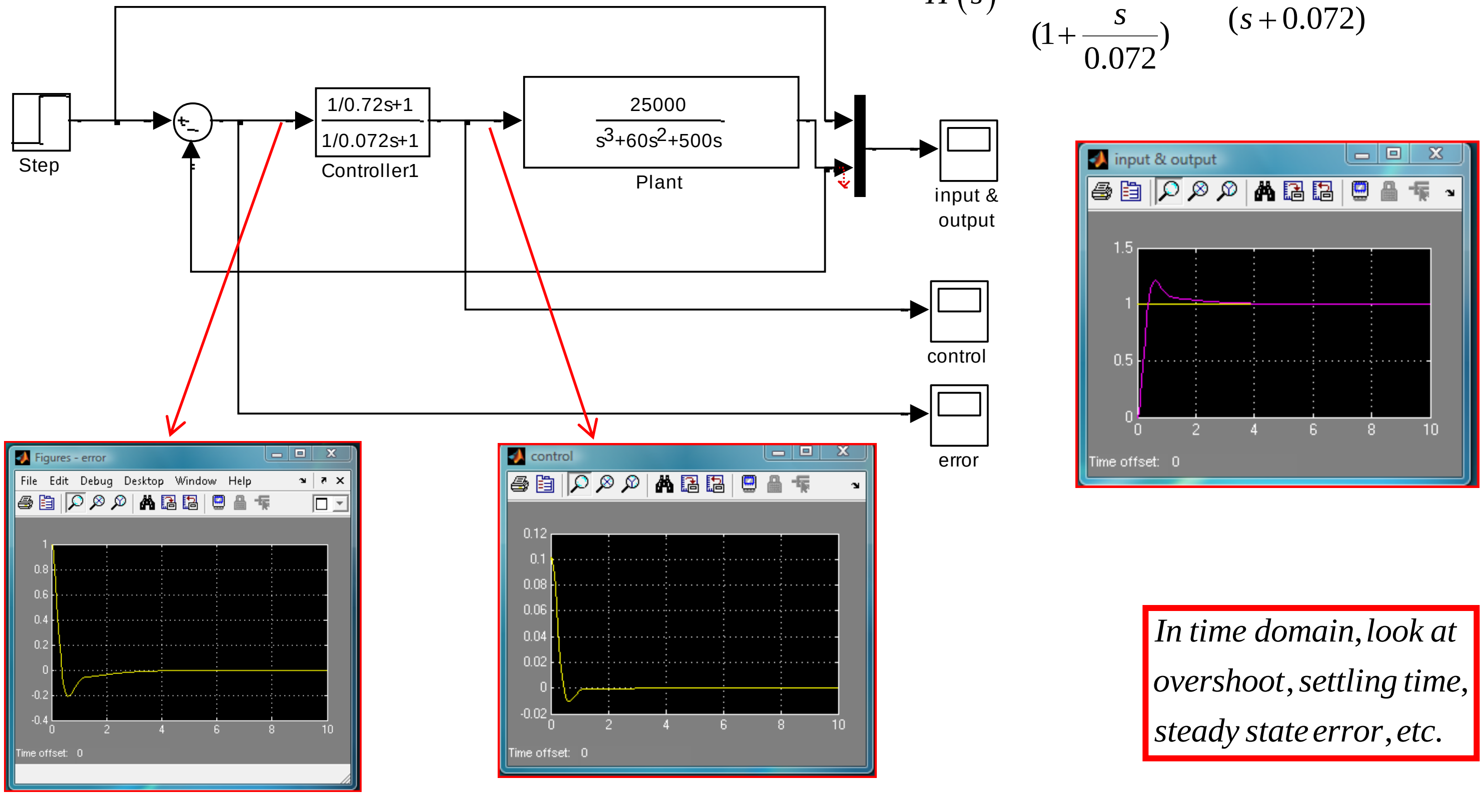
$\phi_m \sim 45^\circ$ to 60°

Output Feedback : Example



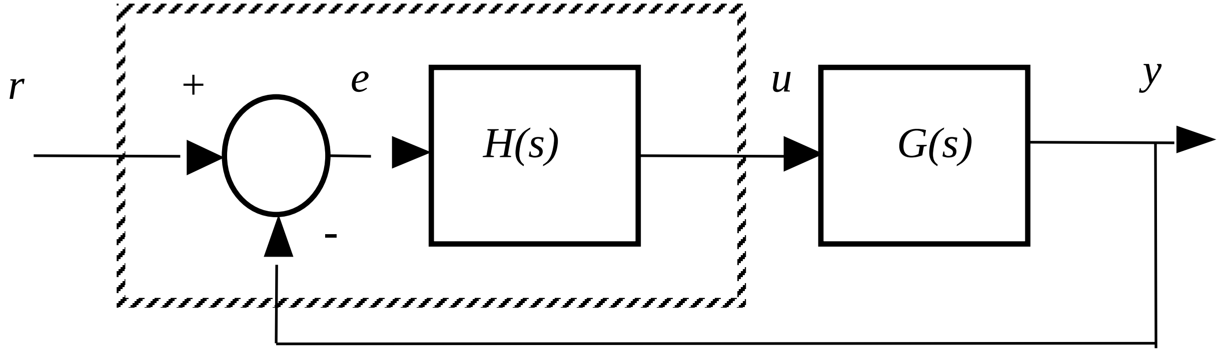
$$G(s)=\frac{25000}{s(s+10)(s+50)}=\frac{50}{s(1+\frac{s}{10})(1+\frac{s}{50})}$$

$$H(s)=\frac{(1+\frac{s}{0.6})}{(1+\frac{s}{0.072})}=\frac{0.144(s+0.6)}{(s+0.072)}$$



In time domain, look at overshoot, settling time, steady state error, etc.

[Bode Plots of Loop Gain, T(s) and S(s)]-1

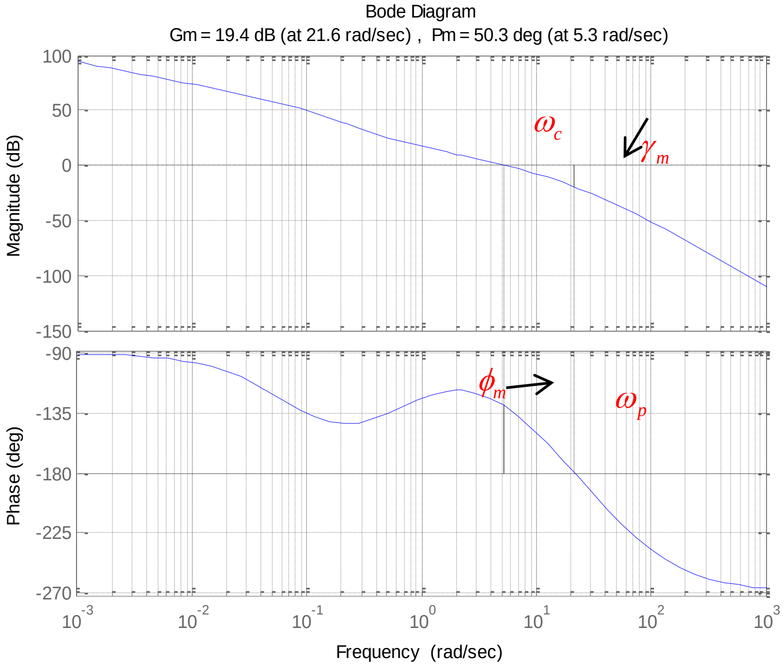


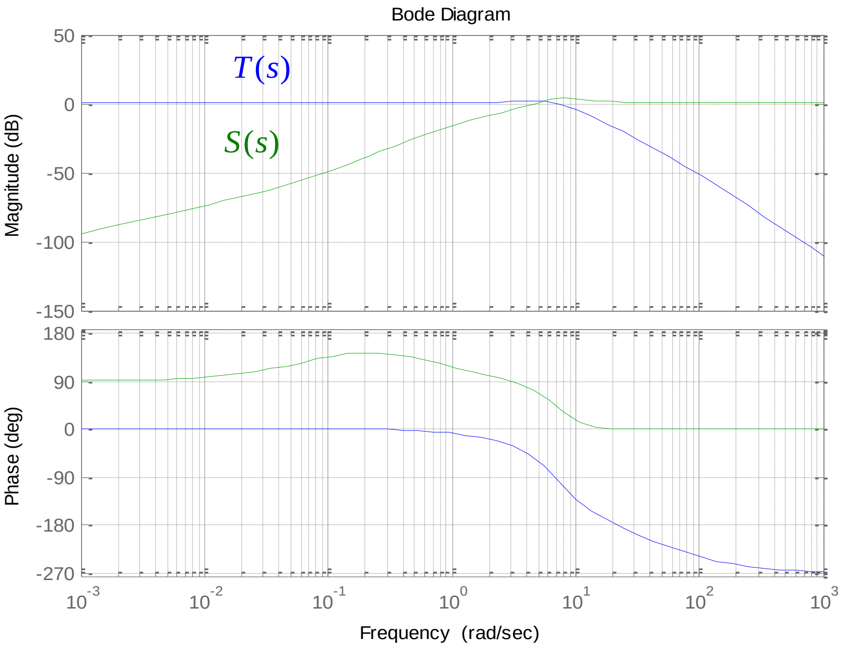
$$G(s)=\frac{25000}{s(s+10)(s+50)}=\frac{50}{s(1+\frac{s}{10})(1+\frac{s}{50})};H(s)=\frac{(1+\frac{s}{0.6})}{(1+\frac{s}{0.072})}=\frac{0.144(s+0.6)}{(s+0.072)}$$

$$T(s)=\frac{G(s)H(s)}{1+G(s)H(s)}=\frac{3000(s+0.6)}{(s+51.4)(s+0.66)(s^2+8.02s+53.1)}$$

$$S(s)=\frac{1}{1+G(s)H(s)}=\frac{s(s+0.072)(s+10)(s+50)}{(s+51.4)(s+0.66)(s^2+8.02s+53.1)}$$

[Bode Plots of Loop Gain, T(s) and S(s)]-2





For margins, plot loop gain, $G(s)H(s)$

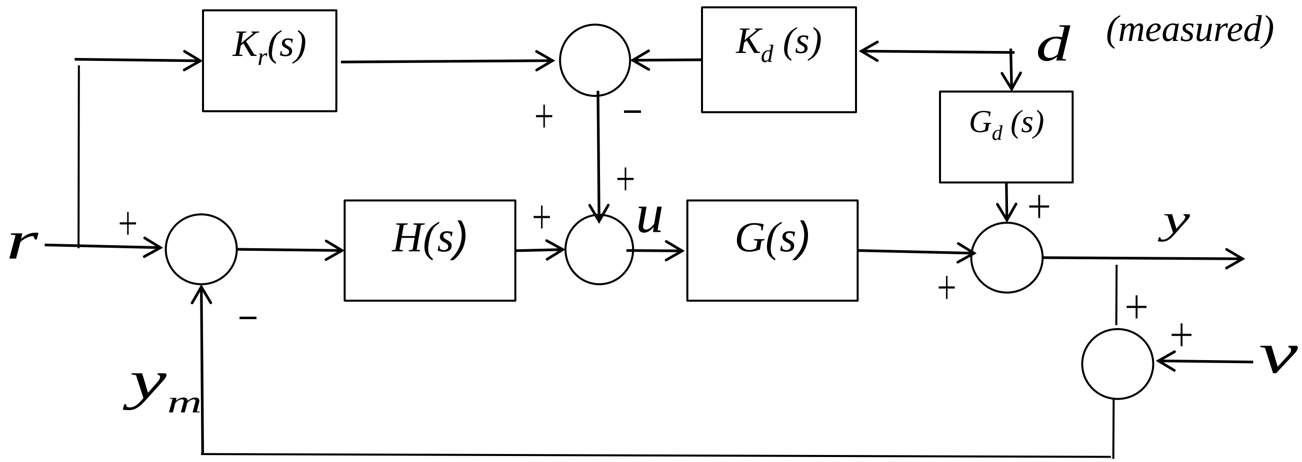
$$G(s)H(s) = \frac{3000(s+0.6)}{s(s+0.072)(s+10)(s+50)}$$

$$T(s) = \frac{3000(s+0.6)}{(s+51.4)(s+0.66)(s^2+8.02s+53.1)}$$

$$S(s) = \frac{s(s+0.072)(s+10)(s+50)}{(s+51.4)(s+0.66)(s^2+8.02s+53.1)}$$

Feedforward Schemes-1

- Feedback and Feedforward scheme (*d* is measured)



$$T(s) = \frac{G(s)H(s)}{1+G(s)H(s)}$$

$$S(s) = \frac{1}{1+G(s)H(s)}$$

Feedforward Schemes-2

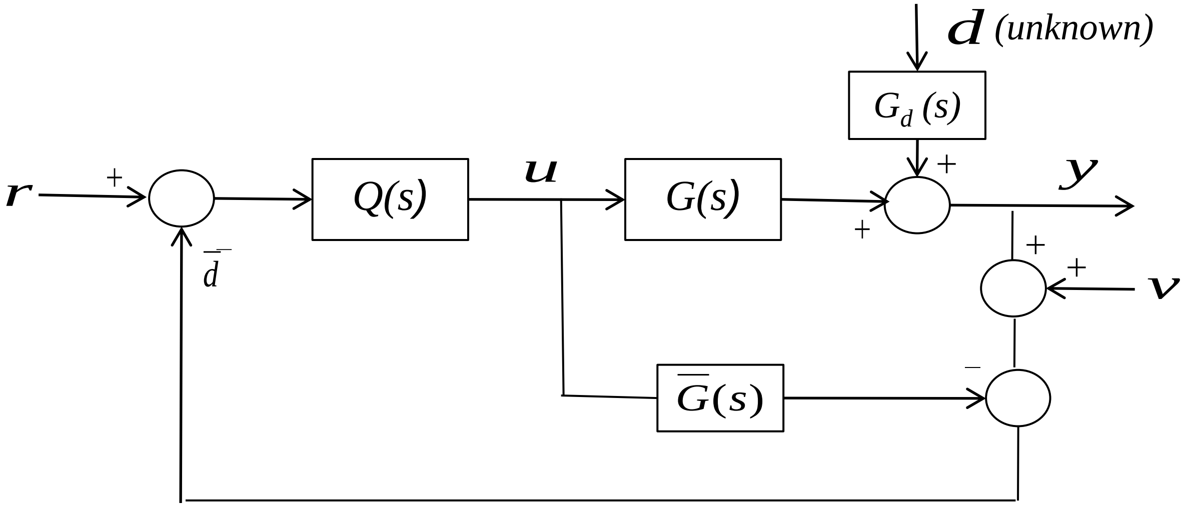
$$\begin{bmatrix} y(s) \\ e(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} T(s)+S(s)G(s)K_r(s) & S(s)[G_d(s)-G(s)K_d(s)] & -T(s) \\ S(s)[1-G(s)K_r(s)] & -S(s)[G_d(s)-G(s)K_d(s)] & T(s) \\ [H(s)+K_r(s)]S(s) & -[H(s)S(s)G_d(s)+S(s)K_d(s)] & -H(s)S(s) \end{bmatrix} \begin{bmatrix} r(s) \\ d(s) \\ v(s) \end{bmatrix}$$

- Feedforward control helps at frequencies where $|S(j\omega)|>1$
- For command tracking, make $S(s)[1-G(s)K_r(s)]$ small $\Rightarrow K_r(s) = \dfrac{1}{G(s)}$
- For disturbance rejection, make $S(s)[G_d(s)-G(s)K_d(s)]$ small $\Rightarrow K_d(s) = \dfrac{G_d(s)}{G(s)}$
- Problems: 1) What if $G(s)$ is not stable?, 2) What if $\dfrac{1}{G(s)}$ is non-causal?

  3) G(s) is not known perfectly,4) Disturbances are never known exactly

Internal Model Control (IMC)-1

- Widely used in Process Control (unknown disturbances, inaccurate models, constraints)
- A convenient theoretical framework for PID tuning rules, Smith predictor, non-minimum phase behavior,…



$$T(s) = \frac{G(s)H(s)}{1+G(s)H(s)}$$

$$S(s) = \frac{1}{1+G(s)H(s)}$$

How is $H(s)$ related to $Q(s)$?

How is $T(s)$ related to $Q(s)$?

Internal Model Control (IMC)-2

$$\text{Recall } y(s) = T(s)[r(s) - v(s)] + G_d(s)S(s)d(s)$$

$$\text{Here } y(s) = \frac{G(s)Q(s)}{1 + Q(s)[G(s) - \overline{G}(s)]}[r(s) - v(s)] + \frac{[1 - \overline{G}(s)Q(s)]G_d(s)}{1 + Q(s)[G(s) - \overline{G}(s)]}d(s)$$

$$T(s) = \frac{G(s)Q(s)}{1 + Q(s)[G(s) - \overline{G}(s)]} \Rightarrow Q(s) = \frac{H(s)}{1 + \overline{G}(s)H(s)} \text{ or } H(s) = \frac{Q(s)}{1 - \overline{G}(s)Q(s)}$$

$$\text{When } \overline{G}(s) = G(s), T(s) = \overline{G}(s)Q(s) \text{ and } S(s) = 1 - \overline{G}(s)Q(s) \Rightarrow Q(s) = H(s)S(s)$$

$$\Rightarrow \text{Both } T(s) \text{ and } S(s) \text{ are linear in } Q(s)$$

$$\Rightarrow y(s) = \overline{G}(s)Q(s)[r(s) - v(s)] + G_d(s)[1 - \overline{G}(s)Q(s)]d(s) = \overline{G}(s)Q(s)[r(s) - v(s)] + G_d(s)S(s)d(s)$$

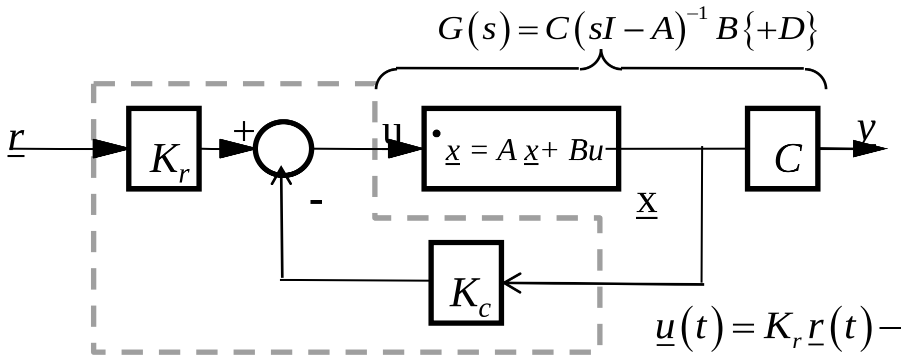
$$\overline{d}(s) = [G(s) - \overline{G}(s)]u(s) + v(s) + G_d(s)d(s)$$

$$\text{if } G(s) = \overline{G}(s) \;\&\; d = v = 0, \overline{d} = 0$$

$$\Rightarrow \text{open-loop}$$

$$\overline{d} \neq 0 \Rightarrow \text{filter this signal}$$

State Variable Feedback



If all states are not measured, need to estimate them and use the estimates for feedback. This can result in poor stability margins. In order to recover robustness, loop transfer recovery (LTR) procedures are used.

$\underline{u}(t) = K_r \underline{r}(t) - K_c \underline{x}(t)$; $K_c = m \times n$ gain matrix

- Design objective is to have $\underline{y}(t) \rightarrow \underline{r}(t)$, especially when $\underline{r}(t)$ = step input, and to have desirable closed-loop stability/transient response. CL system dynamics are:

$$\dot{\underline{x}}(t) = (A - BK_c)\underline{x}(t) + BK_r \underline{r}(t)$$

$$\underline{y}(t) = C\underline{x}(t) + \{D\underline{u}(t)\}$$

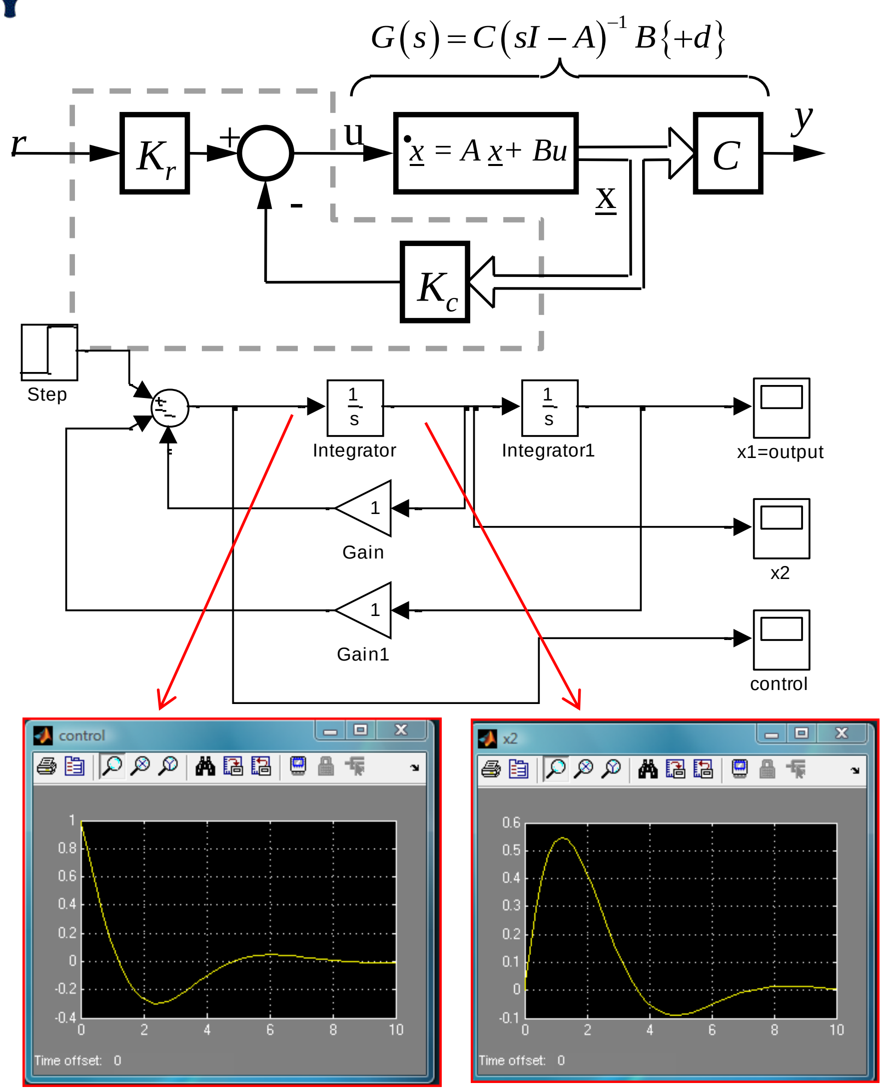
- Selection of feedback gains K_c so that eigenvalues of $A - BK_c$ are in suitable locations in the left hand s-plane and satisfy/optimize certain criteria

  $\Rightarrow$ **pole placement (eigen structure assignment), *LQR*, LQG, H_2, H_∞, μ-synthesis, l_1,…**
- Crossover frequency and phase margin are evaluated by examining the Bode plot of

$$\text{Loop gain} = K_c (sI - A)^{-1} B \Big|_{s = j\omega}$$

Continuous controllers require continuous (i.e., analog) feedback of *y(t)* and/or $\underline{x}(t)$ and implementation using analog components (e.g., circuits, op-amps, analog chips).

State Variable Feedback: SISO Example

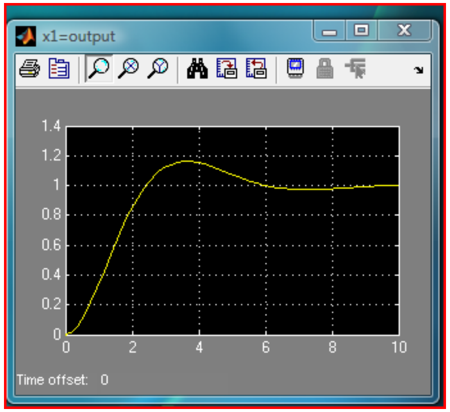


$$G(s) = \frac{1}{s^2}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$K_r = 1; K_c = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$u(t) = K_r r(t) - K_c \underline{x}(t); \quad K_c = 1 \times n \ \text{gain matrix}$$



$$loop\ gain, K_c(sI - A)^{-1}B = \frac{s+1}{s^2}$$

$$Gain\ Margin, \gamma_m = \infty\ db$$

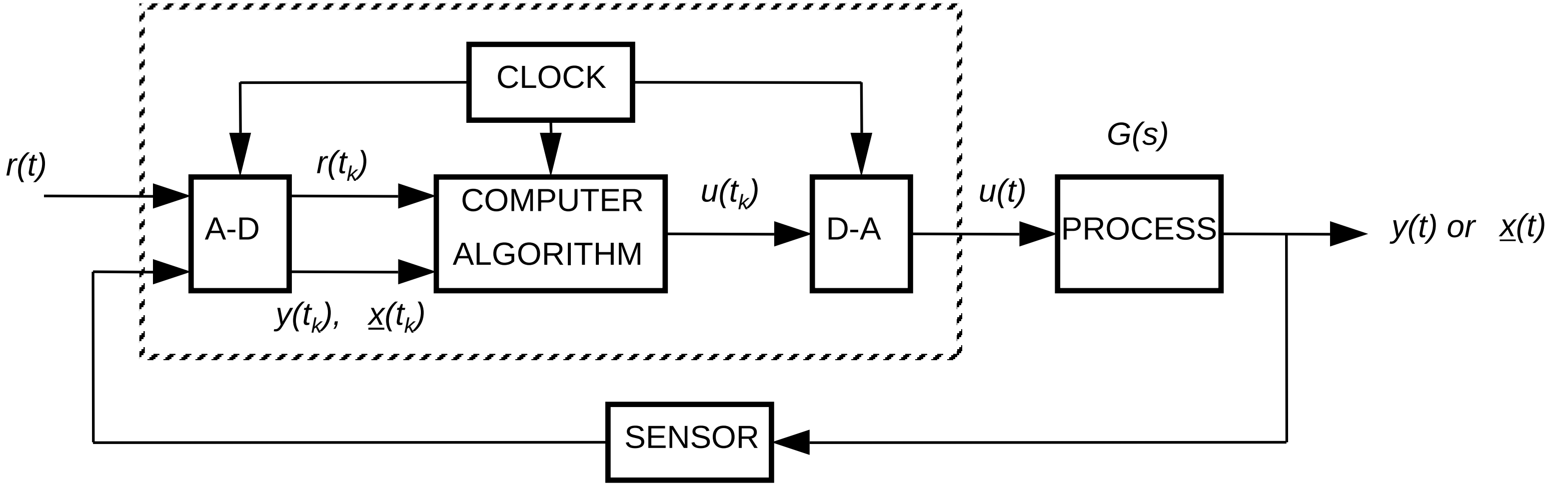
$$Phase\ Margin, \phi_m = 51.8^0$$

Classification of Controller Design Techniques

- Linear Time-invariant Systems
  1. Loop gain shaping
     - Classical designs (lead, lag, lead-lag, PID, feedforward-feedback,)
     - Loop-shaping to minimize sensitivity
  2. Closed-loop transfer function shaping ($T(s)$, $S(s)$, $H(s)S(s)$)
     - Internal Model Control (IMC)
     - Minimize Mixed Weighted Sensitivity (H_∞ optimal control)
  3. State variable feedback controllers
     - Pole placement, LQR, LQG, H_2, H_∞, μ-synthesis, l_1- robust control,..
  4. Numerical optimization-based design
     - Linear Matrix Inequalities (LMI), Model Predictive Control (MPC)

- Non-linear Systems
  - Gain scheduling, MPC, Sliding mode control, Fuzzy control, Neural control

SISO Digital Control Basic Structure-1



We are now dealing *not* with continuous signals in the controller, but with samples of these signals. Usually $t_k = kh$ where ***h*** is the sample time interval. The (real-time) clock maintains synchronism.

SISO Digital Control Basic Structure-2

- Primary steps in computing *u(t)*:

1. Wait for interrupt at time t_k.

(A-D) 2. Sample *r(t)* and *y(t)* to obtain

$y(k) \triangleq$ value of $y(t)$ at time $t = t_k$

$r(k) \triangleq$ value of $r(t)$ at time $t = t_k$.

3. Compute *u(k)*,

$u(k) = H[y(k), y(k-1), \ldots, r(k), \ldots, u(k-1), u(k-2), \ldots]$ This takes ε seconds.

(D-A) 4. Output *u(k)* through the D-to-A converter to give *u(t)*.
If the D-A is a hold circuit then *u(k)* = value of the control over the time interval $[t_k + \varepsilon, t_k + h + \varepsilon]$ where ε is the computational delay at step *3*.

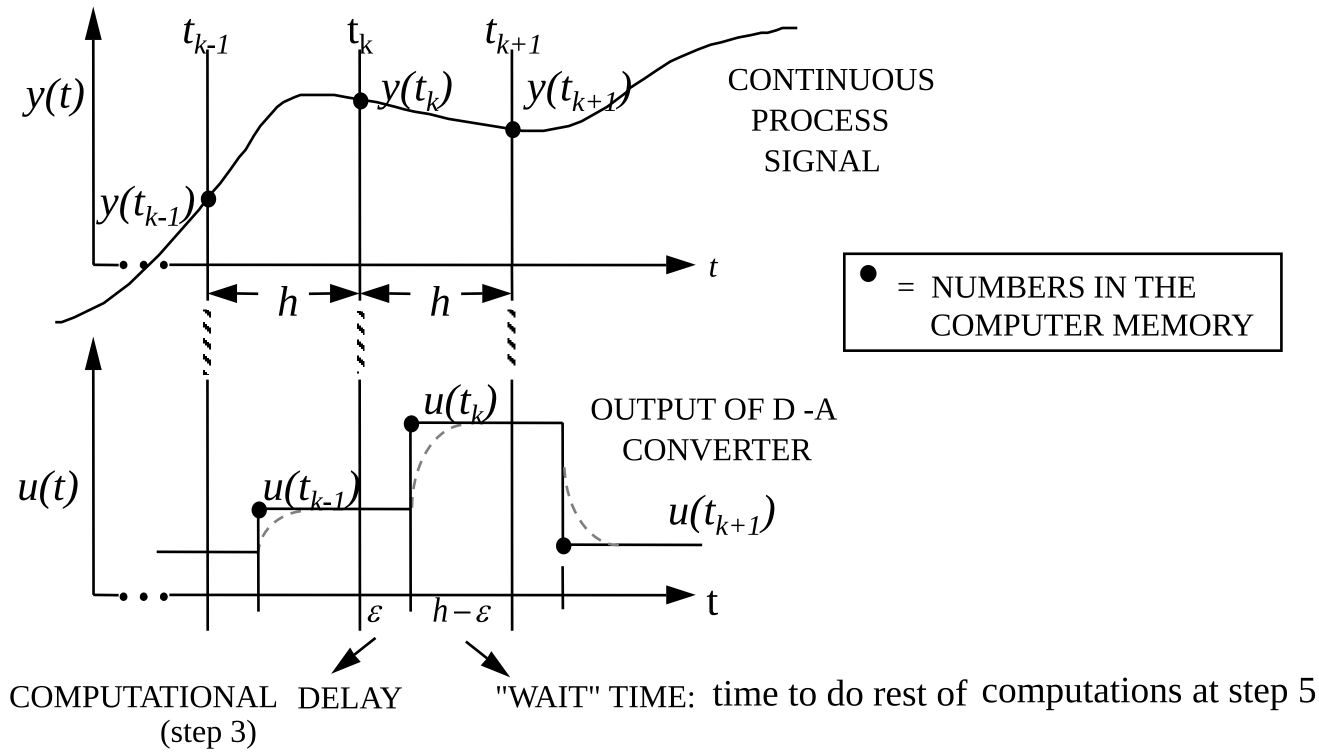
5. Precompute any variables needed for the next cycle.

6. Return to step *1* with $k = k + 1$.

All operations in steps 2-5 must be done in $< h$ *sec!*

Relationship of Time Signals and Samples

There is often a computational delay at step 3, depending on the complexity of the computations performed.



The delay ε is like a delay in $y(t)$ or in the process, i.e., $G(s) \rightarrow G(s)e^{-\varepsilon s}$. Delays reduce ϕ_m!

It is important to minimize the delay at step *3* by arranging things so as to do the least amount of computations at step *3*, while shifting the rest to step *5*.

(A Typical Algorithm For SISO H)-1

$$e(k) = r(k) - y(k) = \text{error at time } t_k$$

$$u(k) = -\alpha_1 u(k\text{-}1) - \alpha_2 u(k\text{-}2) - \cdots - \alpha_m u(k\text{-}q) + \beta_0 e(k) + \beta_1 e(k\text{-}1) + \cdots + \beta_m e(k\text{-}q)$$

This is a <u>difference equation</u>, i.e., a relationship between a sequence of values. An alternate way of writing the algorithm is via a discrete transfer function. Notationally, $u(k) \rightarrow u(z)$, $u(k\text{-}i) \rightarrow z^{-i} u(z)$. Where z^{-1} is the unit shift, or unit delay, operator.

Referred to as an "q^{th} order compensator".

$$u(z) = H(z)e(z)$$

$$H(z) = \frac{\beta_0 + \beta_1 z^{-1} + \cdots + \beta_q z^{-q}}{1 + \alpha_1 z^{-1} + \cdots + \alpha_q z^{-q}}$$

(A Typical Algorithm For SISO H)-2

Implementation of Eq. (1.15):

1. Directly as shown at step 3. This would involve ~ $(2q + 1)$ MADDS.

   or

2. Compute $u(k) = WI + \beta_0 e(k)$ *at step* 3

   $$\text{where } WI = -\sum_{i=1}^{q} \alpha_i u(k-i) + \sum_{i=1}^{q} \beta_i e(k-i)$$

   was computed at step 5 during the previous time step.
   Requires only *1* MADD!

   => Clever organization of the algorithm can reduce ε.

Diffs. in Digital vs. Analog Control Methods

- Control design based on <u>samples</u> of *y(t)*, *r(t)*, *<u>x</u>(t)*
- Control input to system is piecewise constant over intervals of length *h* (assuming D -A is a hold circuit)
- Computational delays
- Controlling a <u>continuous</u> system, *G(s)*, using a <u>discrete</u> algorithm, *H(z)*
  => a mix of continuous and discrete elements constitutes the FB loop
- Most analysis will need to be performed using z-transforms and working in the *z*-plane
- Computer-aided design software becomes much more necessary for analysis, design, evaluation
- Effects of round-off error in computations due to finite word length
- Quantization error in A-D conversion (not a major issue with 16 and 24-bit A-D conversion)

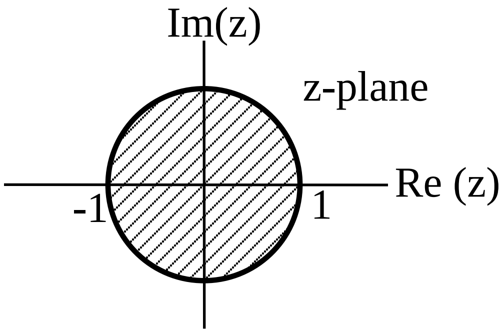
Stability of the Time Response

p(z) = denominator of transfer function = characteristic polynomial

$$= z^n + a_1 z^{n-1} + \cdots + a_n$$

$$= (z-\lambda_1)(z-\lambda_2)\cdots(z-\lambda_n); \quad \lambda_i = roots\ of\ p(z)$$

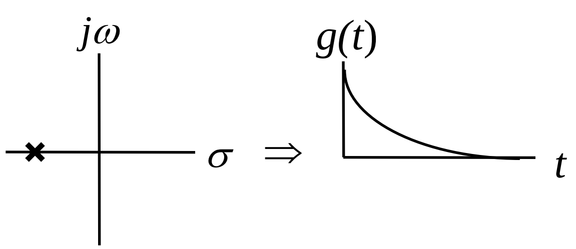
If input $u(k) \not\to \infty$, then $y(k) \not\to \infty$ if roots λ_i of the characteristic polynomial have $|\lambda_i| < 1$.

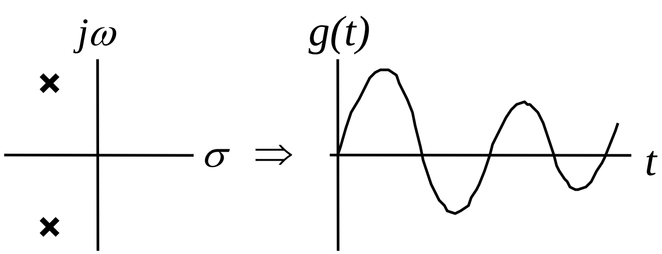


Bounded Input and Bounded Output (BIBO) Stability

λ_i are called the <u>poles</u> of the system. We say a system is <u>stable</u> if and only if λ_i lie within the unit circle.

Usually the difference equation (1.19) is a discrete model of an underlying <u>continuous</u> process or continuous signal. We often have a good mental picture of the impulse response *g(t)* given the poles s_i of the continuous transfer function *G(s)*.



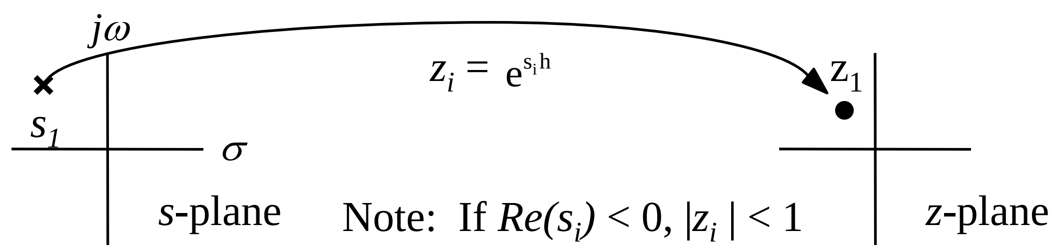


How do we develop the same insight in the *z*-plane?

Continuous vs. Discrete Relationships

$z = e^{sh}$ gives relationship between the Laplace (s) and z domains

Ex. If $f(t)$ has a Laplace transform with poles @ $s_1, s_2, \ldots$ then the sampled $f(kh)$ has a z-xform with poles at $z_1 = e^{s_1 h}$, $z_2 = e^{s_2 h}$, ...



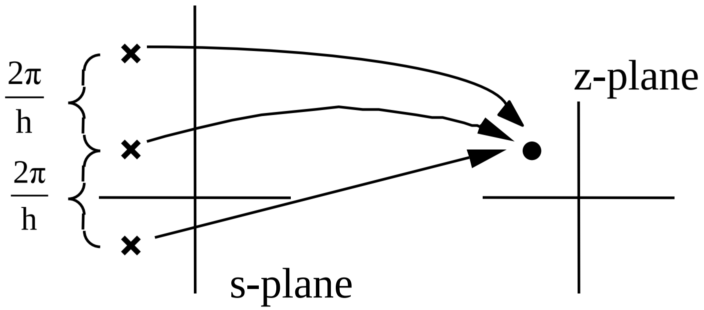
- Non-uniqueness property:

$$e^{(\sigma + j\omega)h} = e^{\left(\sigma + j\omega \pm j\frac{2\pi}{h}N\right)h} \quad N = \text{integer}$$

$$\Rightarrow s = \sigma + j\omega \text{ and}$$

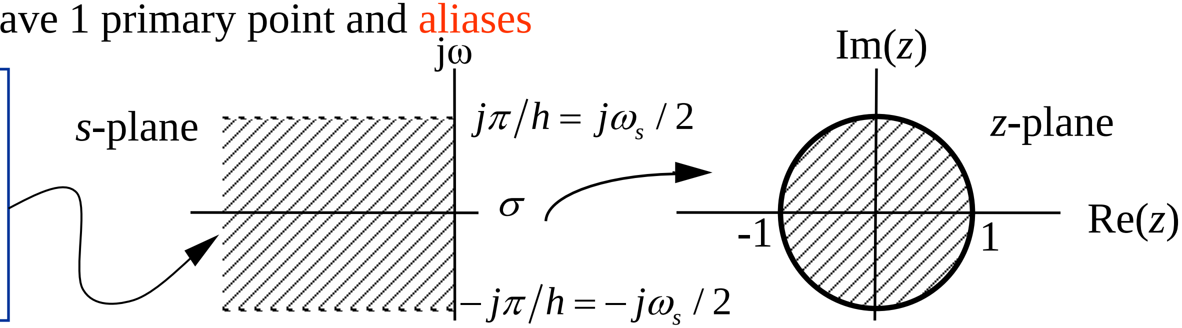
$$\sigma + j\omega \pm j\frac{2\pi}{h}N$$

gives same z

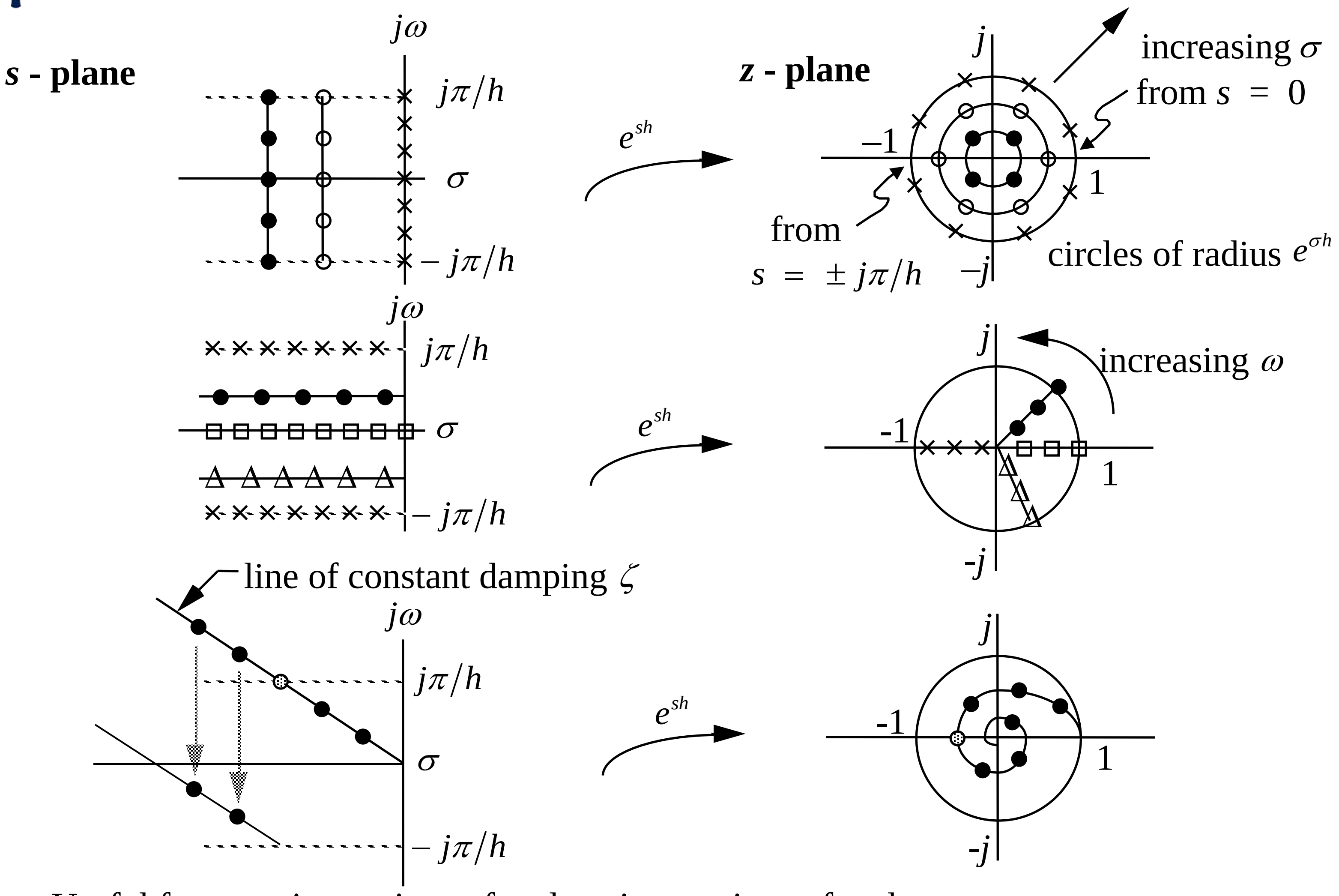


not a 1-1 mapping -- have 1 primary point and aliases

Primary strip -- these points will fill the unit circle. Points outside of this strip will be duplicates.



s – to – z Plane Mapping



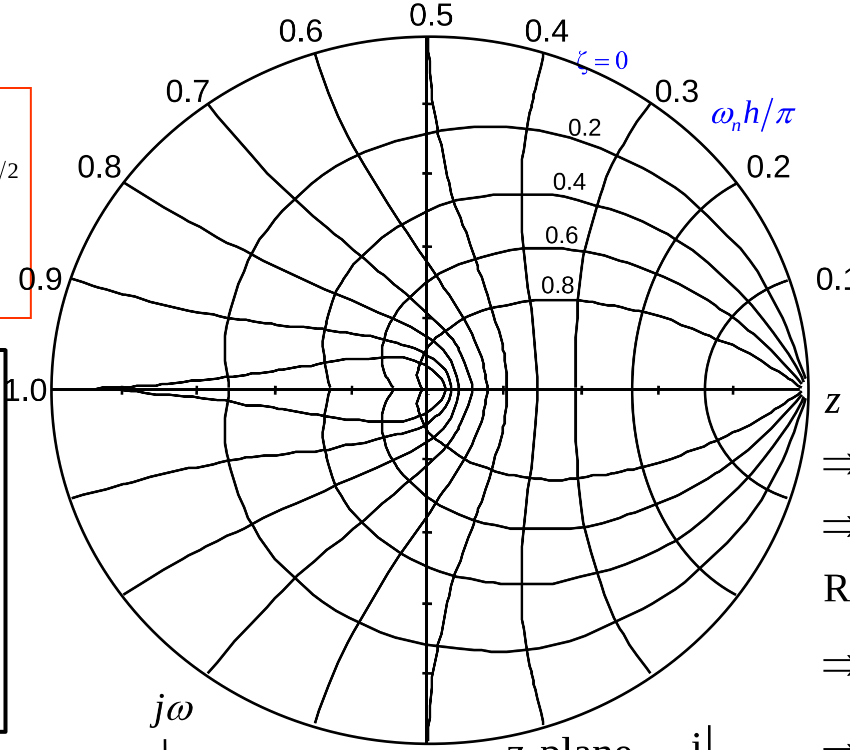
Useful for mapping regions of s-plane into regions of z-plane

Loci of Constant Damping Ratio (ζ) and Natural Frequency (ω_n) in s-Plane to z-Plane Mapping

If $z = re^{j\theta}$, then:

$$\zeta = -\ln r\left[\ln^2 r + \theta^2\right]^{-1/2}$$

$$\omega_n h = \left[\ln^2 r + \theta^2\right]^{1/2}$$



Z-grid

- For a given ζ, spirals are symmetric around real axis
- For a given ζ
  $$\theta = \pm\frac{\sqrt{1-\zeta^2}}{\zeta}\ln r$$
- For a given θ,
  $$\zeta \uparrow \Rightarrow |\ln r| \uparrow \Rightarrow r \downarrow$$

$$z = re^{j\theta} = e^{sh} = e^{-\sigma h}e^{j\omega_d h}$$

$$\Rightarrow -\sigma h + j\omega_d h = \ln r + j\theta$$

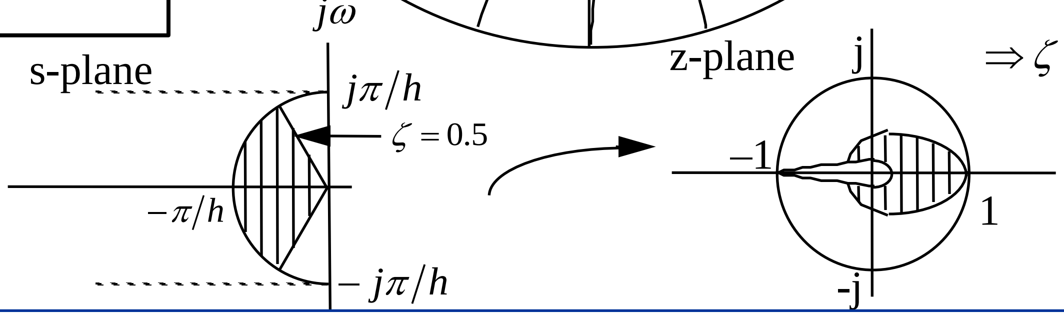
$$\Rightarrow \sigma h = -\ln r \;\&\; \omega_d h = \theta$$

$$\text{Recall } \omega_n^2 h^2 = (\sigma^2 + \omega_d^2)h^2$$

$$\Rightarrow \omega_n h = \sqrt{(\ln r)^2 + \theta^2}$$

$$\Rightarrow \zeta = \frac{\sigma h}{\omega_n h} = \frac{-\ln r}{\sqrt{(\ln r)^2 + \theta^2}}$$

Ex.



Note: The semi-circle of radius π/h fills the entire unit circle minus the small "airfoil" area on z-axis.

Summary

1. Feedback Control Structures for Continuous-time Systems
   - Output feedback (series compensation, Proportional-Integral-Derivative (PID), Feed forward-feedback, Internal Model Control…)
   - State variable feedback
2. Classification of Control Design Techniques
3. Digital Control Loop Structure
   - Relationships among time signals
   - Typical algorithm implementation considerations
4. Discrete-time System Stability
5. Continuous-time-vs.-Discrete-time Relationships
   - $s \rightarrow z$ plane mapping