Lecture 4

Performance Criteria, Sensor Rating, Actuator Issues

Prof. Krishna R. Pattipati
Prof. David L. Kleinman

Dept. of Electrical and Computer Engineering
University of Connecticut
Contact: krishna@engr.uconn.edu (860) 486-2890

ECE 6095/4121
Digital Control of Mechatronic Systems

Performance Criteria and the Design Process

1. Design Approach and the Design Process

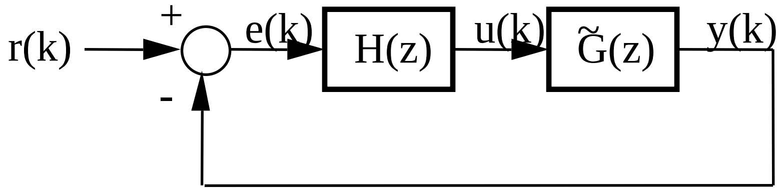
2. Performance Measures and Criteria
   - Stability and phase margin
   - Steady-state accuracy
   - Max peak criteria
   - Speed of response/transient, sum of absolute error, sum of square error
   - Sensitivity and return difference
   - Sensor rating parameters
   - Accuracy versus Precision
   - Actuator nonlinearities
   - Bandwidth design

3. Simulation of Closed-Loop Time Response
   - Simulation program structure
   - Control algorithm simulation
   - Modifications to simulate time delay

\

Elements of Feedback System Design I

- Series Compensator Design Structure ("Classical")



Loop gain = $\tilde{G}(z)\,H(z)$

Given $\tilde{G}(z)$ design a suitable $H(z)$

$$u(z) = H(z)\ [r(z) - y(z)]$$

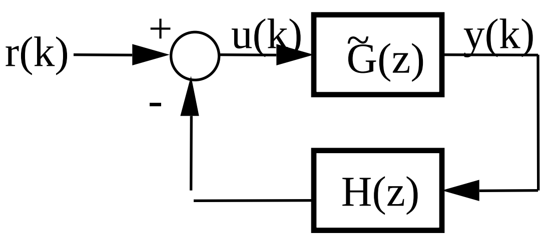
Closed-loop transfer function

$$\frac{y(z)}{r(z)} = T(z) = \frac{\tilde{G}(z)H(z)}{1+\tilde{G}(z)H(z)} \;;$$

Closed-loop characteristic polynomial

$$e(z) = \frac{1}{1+\tilde{G}(z)H(z)}\,r(z)$$

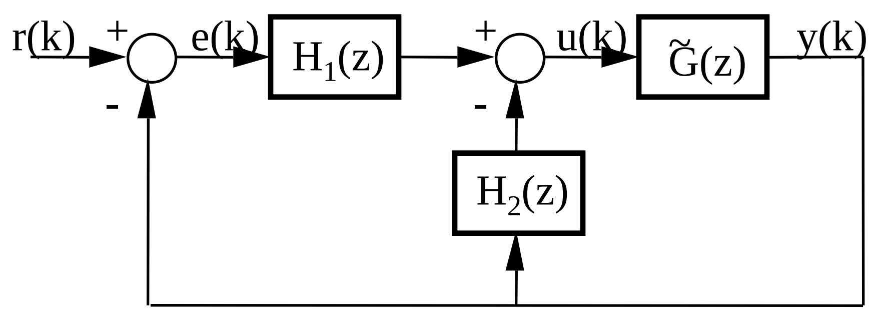
- Alternate loop structures
  - Feedback compensator design



$$u(z) = r(z) - H(z)\ y(z)$$

$$T(z) = \frac{\tilde{G}(z)}{1 + \tilde{G}(z)\ H(z)}$$

  - Mixed series/feedback compensator design



Most general form, allows for FB using e and/or y.

Elements of Feedback System Design II

- State Variable Design Structure ("Modern")

  Given $\underline{x}(k+1) = \Phi \underline{x}(k) + \Gamma u(k)$, $y(k) = C\underline{x}(k)$

  design suitable K, K_r, $u(k) = K_r\, r(k) - K\underline{x}(k)$

  Closed-loop:

$$\underline{x}(k+1) = \overbrace{(\Phi - \Gamma K)}^{\bar{\Phi}}\underline{x}(k) + K_r \Gamma\, r(k)$$

$$\frac{y(z)}{r(z)} = T(z) = K_r C(zI - \Phi + \Gamma K)^{-1}\Gamma = \frac{K_r N(z)}{|zI - \Phi + \Gamma K|}$$

  Alternate Formula:

$$T(z) = \frac{\overbrace{K_r C(zI - \Phi)^{-1}\Gamma}^{\tilde{G}(z)}}{1 + K(zI - \Phi)^{-1}\Gamma}$$

  Derivation:

  (1) $\underline{x}(z) = (zI - \Phi)^{-1}\, \Gamma\, u(z)$

  (2) $u(z) = K_r r(z) - K(zI - \Phi)^{-1}\, \Gamma\, u(z)$

  (3) $u(z) = [\, 1 + K(zI - \Phi)^{-1}\, \Gamma\,]^{-1}\, K_r r(z)$

  (4) Substitute into $y(z) = C(zI - \Phi)^{-1}\, \Gamma\, u(z)$

  => Closed-loop characteristic polynomial is

$$p(z) = |zI - \Phi + \Gamma K| \quad \text{or} \quad p(z) = 1 + K(zI - \Phi)^{-1}\Gamma$$

- Optimal Control Design ("Classy")

  One method for obtaining K, K_r -- by optimizing some criterion.

Design Approaches to be Considered

- For series compensation design of H(z)

$$H(z) = \frac{\beta_0 z^m + \beta_1 z^{m-1} + \cdots + \beta_m}{z^m + \alpha_1 z^{m-1} + \cdots + \alpha_m} \quad (\text{m-th Order Compensator})$$

  (1) Discretization of a continuous design

$$H(s) \rightarrow \tilde{H}(z)$$

  where H(s) is a series compensator designed for G(s) (will usually be OK when h is very small).

  (2) Direct design methods for H(z) given $\tilde{G}(z)$.

- For SVFB design of K, K_r

  (1) Discretization of continuous design gains

  $K \rightarrow \tilde{K}; \;\; K_r \rightarrow \tilde{K}_r$ where K, K_r were designed for $\dot{\underline{x}} = A\underline{x} + Bu$.

  (2) Pole placement, direct design methods

  Select K so that $|zI - \Phi + \Gamma K|$ has desired roots.

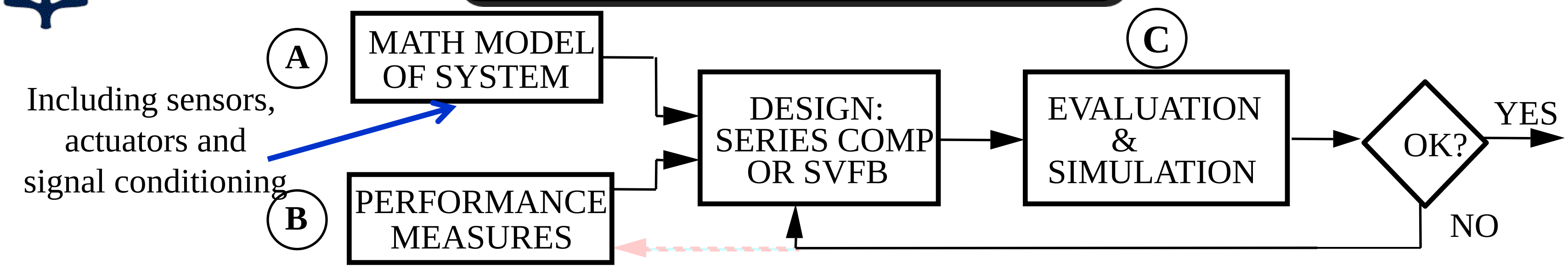
  (3) Optimization methods

  Find $u(k) = K_r r(k) - K\underline{x}(k)$ to optimize some performance criterion ==> K^*, K_r^*.

- Methods for state estimation when $\underline{x}(k)$ is not directly measurable,

  $\{y(\bullet)\} \rightarrow \hat{\underline{x}}(k)$.

The Design Process



A) <u>Mathematical Model of System to be Controlled</u>

Defined by discrete equivalent $\tilde{G}(z)$, or $\{\Phi, \Gamma, C\}$

B) <u>Performance Measures and Concerns</u>

Mathematical criteria that are driven by customer's qualitative/quantittaive specifications for behavior of the closed-loop system.

(1) Stability of the closed-loop system
- A property of loop dynamics not of *r(k)*
- Without stability cannot discuss much else

(2) Steady-state accuracy
- Does $\underline{y}(k) \rightarrow \underline{r}(k)$ as $k \rightarrow \infty$
- If $\underline{r}(k) = 0$ desire $\underline{y}(k)$ & $\underline{x}(k) \rightarrow \underline{0}$ for all $\underline{x}(0)$
- Resonant peak, Bandwidth, cutoff (roll-off) rate

(3) Speed of response/transient, sum of absolute error (SAE), sum of squared error (SSE)
- Transient response linked to CL pole locations

(4) Sensitivity/robustness
- Ability of CL system to perform with $\Delta\tilde{G}(z)$, $\Delta\tilde{G}_d(z) \Rightarrow$ bounds on *Loop gain*
- Feedback desensitizes loop to variations in $\tilde{G}(z)$, $\tilde{G}_d(z)$

(5) Sensor rating parameters

6) Accuracy versus Precision

7) Actuator nonlinearities

8) Bandwidth design

1 - Stability of the Closed-Loop System

- Roots of closed-loop characteristic polynomial $p_{CL}(z)$ in unit circle $(|\lambda_i| < 1)$

roots eig

$$SISO: 1 + G(z)H(z) = 0 \text{ or } \begin{cases} |zI - \Phi + \Gamma K| = 0 \textit{ valid for SISO or MIMO} \\ 1 + K(zI - \Phi)^{-1}\Gamma = 0 \textit{ for SISO} \end{cases}$$

- Will need a simple test to determine if a polynomial p(z) has any roots with $|\lambda| \geq 1$.
  - Recall Routh test for whether p(s) has roots in RHP…… **Jury** test
- Phase margin ϕ_m used to give degree of stability. "How much more negative phase shift (phase lag) can you put in the FB loop before the system becomes unstable?" ~ tolerance to time-delay.

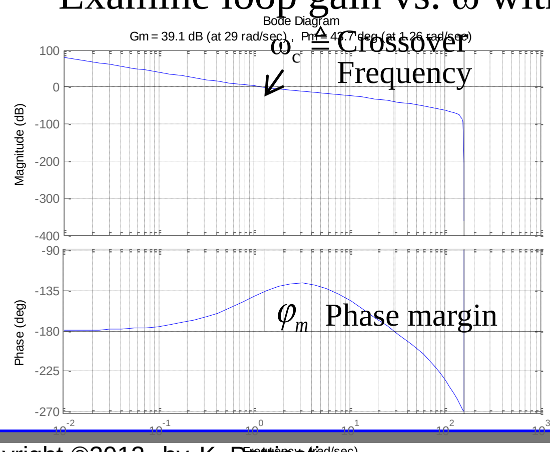
To determine ϕ_m, use Bode (or Nyquist, or Nichols) plot of loop gain of SISO system:

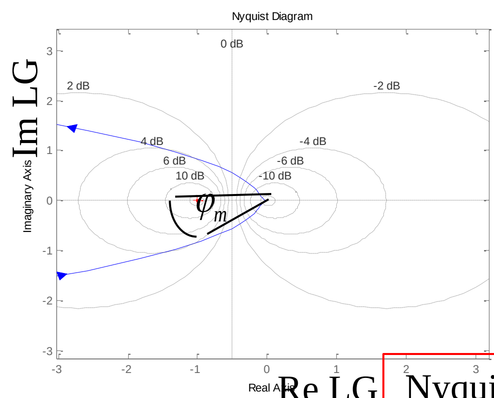
- $LG_{ain} = \tilde{G}(z)\,H(z)$ series compensation
- $LG_{ain} = K(zI - \Phi)^{-1}\,\Gamma$ for SVFB

$$A, B \xrightarrow{\text{c2d}} \Phi, \Gamma \xrightarrow{\text{ss2tf}} \frac{N(z)}{D(z)} \xrightarrow{\text{margin}} \phi_m, \omega_c$$

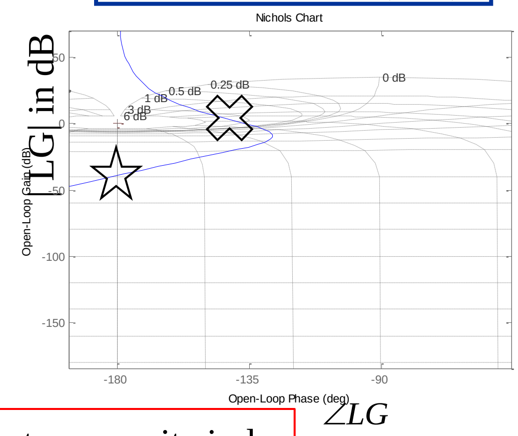
$$\phi_m = \pi + \measuredangle LG_{ain}(j\omega_c)$$

Examine loop gain vs. ω with $z = e^{j\omega h}$





Nyquist contour = unit circle



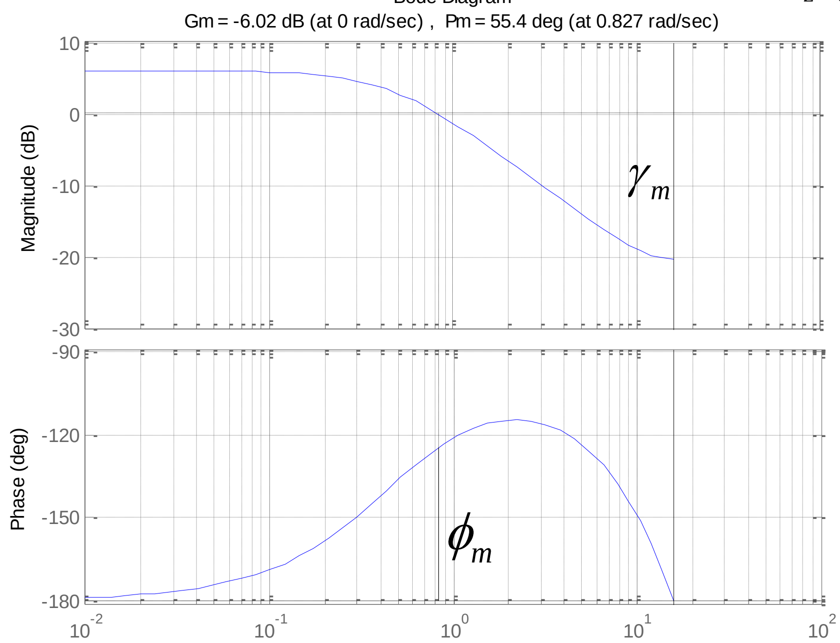
Example

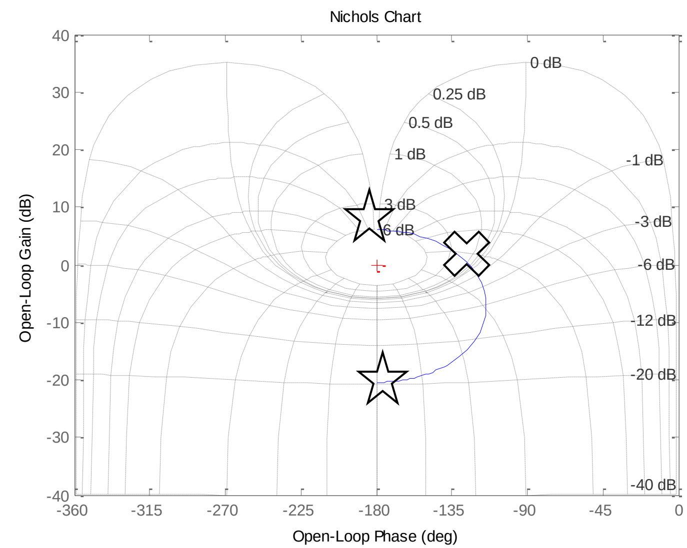
The system $\dot{x}(t) = 0.5\,x(t) + 0.95\,u(t)$, $y(t) = x(t)$ is controlled digitally using the algorithm $u(k) = K_r\,r(k) - x(k)$, with time step $h = 0.2$ sec. Determine the phase margin.
(Note that the open-loop G(s) is unstable.)

i) Discrete equivalent model $\Phi = e^{+0.5h} = 1.1$, $\Gamma = (e^{+0.5h} - 1)\,(0.95/0.5) = +0.2$

ii) Check stability of closed-loop (K = +1) $\Rightarrow$ $\Phi - \Gamma K = 0.9 \Rightarrow$ stable

iii) Obtain ϕ_m via Bode plot of LG= $K(zI - \Phi)^{-1}\Gamma|_{z = e^{j\omega h}} = 0.2/(z - 1.1)$





Analytic approach: find ω_c where $0.2/|\,z - 1.1\,| = 1$ @ $z = e^{j\omega_c h}$
$0.2 = |\,(\cos\omega_c h - 1.1) + j\sin\omega_c h\,| \Longrightarrow 0.04 = (\cos\omega_c h - 1.1)^2 + \sin^2\omega_c h$
solving gives $\cos\omega_c h = 0.986 \Longrightarrow \omega_c = (1/h)\cos^{-1}(0.986) = 0.827$ rad/sec

$\phi_m = 55.4^0$
ω_c=0.827 rad/sec
γ_m=-6dB, 20.4dB
$\Rightarrow$ Stable for (0.5,10.5)

2a – Steady-State Accuracy to a Step Input

- If r(k) is a step input (e.g., commanded change in setpoint) of value A, want $y(k) \to A$ in steady-state (s.s.)
  Final value theorem for y_{ss} - provided CL system is stable:

$$\lim_{k\to\infty} y(k) = (1-z^{-1})T(z)r(z)\big|_{z=1} = (1-z^{-1})T(z)\frac{A}{1-z^{-1}}\bigg|_{z=1} = AT(1) \Rightarrow \underline{T(1)=1}$$

  - For series compensation design only $T(1) = 1 = \dfrac{\tilde{G}(1)H(1)}{1+\tilde{G}(1)H(1)} \Rightarrow \tilde{G}(1)H(1) = \infty$

    Requires loop gain to have a pole at z = 1 $\Rightarrow G(z)H(z) = \dfrac{N(z^{-1})}{(1-z^{-1})D(z^{-1})}$

    => Need an integrator in either G (i.e., $\tilde{G}$) or H

  - For SVFB design achieve T(1) = I via proper choice of K_r (valid for MIMO also)

$$T(z) = C(zI_n - \Phi + \Gamma K)^{-1}\Gamma K_r \Rightarrow K_r = \left[C(I_n - \Phi + \Gamma K)^{-1}\Gamma\right]^{-1}$$

or

$$T(z) = K_r C(zI-\Phi)^{-1}\Gamma[I_m + K(zI-\Phi)^{-1}\Gamma]^{-1}$$
$$\Rightarrow K_r = [I_m + K(I-\Phi)^{-1}\Gamma][C(I-\Phi)^{-1}\Gamma]^{-1}$$

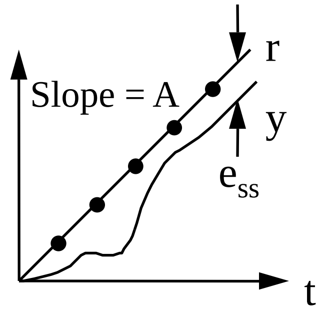
- If $T(1) \neq 1$ there will be a steady-state error, $A - y_{ss}$.

  Fractional error $\triangleq \dfrac{1}{K_p} = \dfrac{A-y_{ss}}{y_{ss}} = \dfrac{1-T(1)}{T(1)} \Rightarrow K_p = \dfrac{T(1)}{1-T(1)}$ usually large

  Steady-state error, $e_{ss} = \dfrac{A}{1+K_p} \sim \dfrac{A}{K_p}$

2b – Steady-State Accuracy to a Ramp Input

- When r(k) is a ramp input, r(k) = Akh, we want to command a rate of change in setpoint.



$y_{ss} \rightarrow \beta kh - \alpha$
- need T(1) = 1 for β = A
(otherwise $e_{ss} \rightarrow \infty$)

- Relative "steady-state" error $\dfrac{e_{ss}}{A} = \dfrac{\alpha}{A} = \dfrac{1}{K_v}$ in seconds
  - in general, $K_v = -\dfrac{1}{h}\left[\dfrac{dT(z)}{dz}\right]^{-1}_{z=1}$
  - can show $\dfrac{1}{K_v} = h\left[\sum \dfrac{1}{1-p_i} - \sum \dfrac{1}{1-z_i}\right]$ poles, zeros of T(z)
- For series compensation structure only

$$K_v = \frac{(1-z^{-1})}{h}\tilde{G}(z)H(z)\Big|_{z=1} = \frac{N(1)}{hD(1)} = \lim_{s\to 0} s\tilde{G}(e^{sh})H(e^{sh})$$

⇒ need at least one integrator in forward loop gain ($\tilde{G}H$) since $s\tilde{G}H \rightarrow K_v$ at low frequency ⇒ $\tilde{G}H \rightarrow K_v/s$ as $s \rightarrow 0$, i.e., K_v is the gain of the Low Frequency asymptote.
- provides criterion for selecting LF loop gain

2c – Steady-state Error to Sinusoidal Inputs

- Series Compensation Design

Error $\quad r(z) - y(z) = e(z) = \dfrac{1}{1 + \tilde{G}(z)H(z)} r(z)$

=> Want $|\tilde{G}(z)H(z)|$ large over the frequency range of interest where $z = e^{j\omega h}$

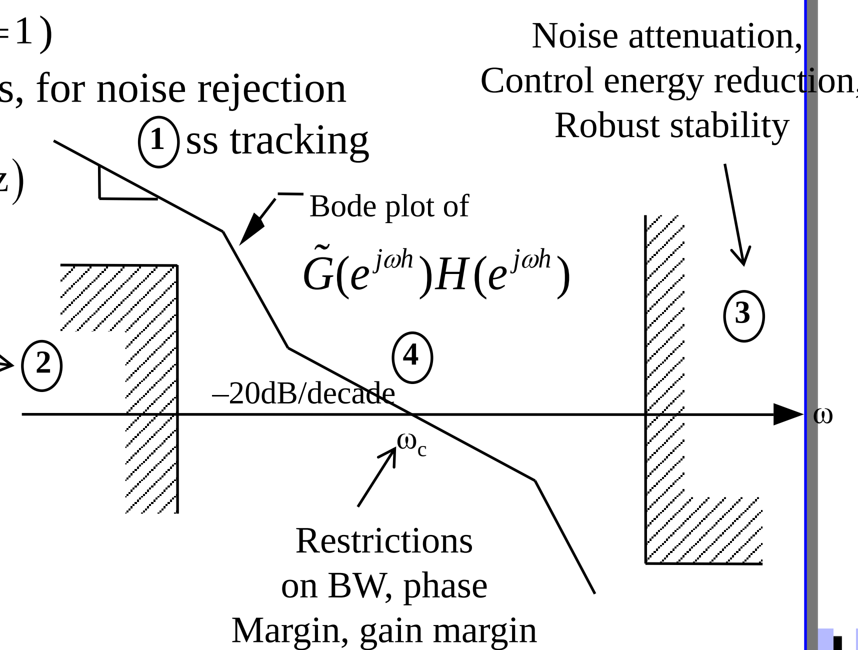
Places lower bounds on ω_c (where $|\tilde{G}H|_{z = e^{j\omega h}} = 1$)

But want $|\tilde{G}(z)H(z)|$ small at high frequencies, for noise rejection

$$y(z) = \frac{\tilde{G}(z)H(z)}{1 + \tilde{G}(z)H(z)} r(z)$$

These provide criteria for selection of H(z).

$\omega_c \sim$ Bandwidth of CL system



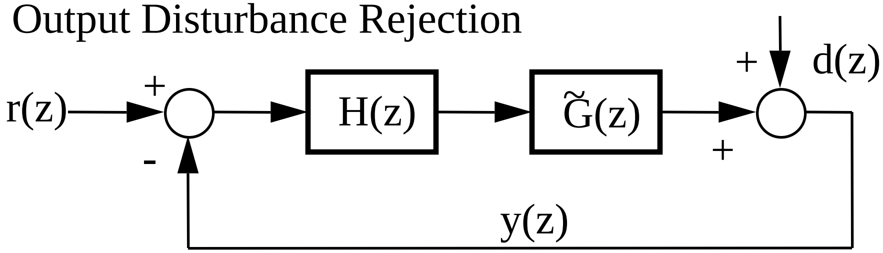
- SVFB Design

Bandwidth determined by CL pole locations

Obtain ω_c via Bode plot of $K(zI - \Phi)^{-1}\Gamma$

=> implicit specification of ω_c

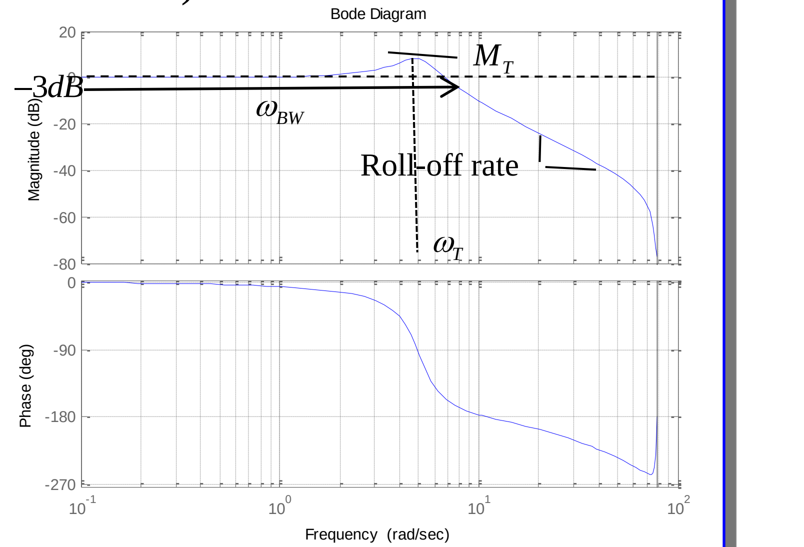
- Output Disturbance Rejection



$$y(z) = T(z)r(z) + \frac{d(z)}{1 + \tilde{G}(z)H(z)}$$

2d – Maximum Peak Criteria

- Closd-loop $T(e^{j\omega h})$ measures: (values for 2nd order system shown)
  - Resonant peak (M_T): $\dfrac{1}{2\varsigma\sqrt{1-\varsigma^2}}$
  - Resonant frequency (ω_T): $\omega_n\sqrt{1-2\varsigma^2}$
  - Bandwidth (ω_{BW}): $\omega_n[(1-2\varsigma^2)+\sqrt{4\varsigma^4-4\varsigma^2+2}]^{1/2}$
  - roll-off (cutoff) rate



- Compression-type piezoelectric accelerometer sensors typically have peak in the frequency response with resonant frequency (typically 20 kHz)
  - useful frequency range = ¼ resonant frequency $\Rightarrow$ typically up to 5 kHz
  - Flat gain curve (less than 1dB (≈12%) change over a decade)

3 – Speed of Response/Transient

- Related to the location of the closed loop (CL) poles and zeros.
- Require some nominal input, i.e., speed of response to "what".
  - Most common test input is unit step, next ramp.
- Examine step response of a 2nd order closed loop system:
  - Many systems are interconnections of 2nd order parts
  - Many systems have a dominant 2nd order pair (roots with smallest Re[s], or largest | z |)
  - Consider T(s), then s→ z plane map to get T(z) poles

- Total variation (TV) $= \sum_{i=1}^{\infty} |v_i|$

$$= \sum_{k=0}^{\infty} |y(k+1) - y(k)| = \sum_{k=0}^{\infty} |g(k)|$$

- *Sum absolute error (SAE)*

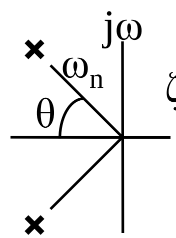
$$SAE = \sum_{k=0}^{\infty} |e(k)|$$

- Sum squared error (SSE)

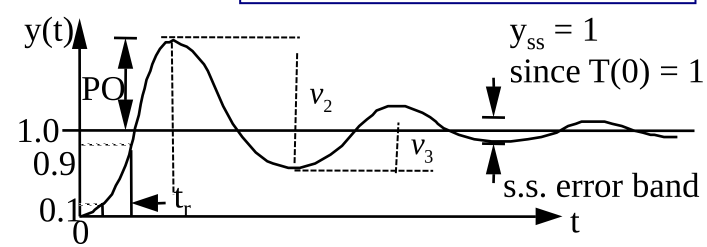
$$SSE = \sum_{k=0}^{\infty} e^2(k)$$

$$(CL)\,T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \qquad 0 < \zeta \leq 1$$

$$\lambda_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$



$\zeta = \cos\theta$



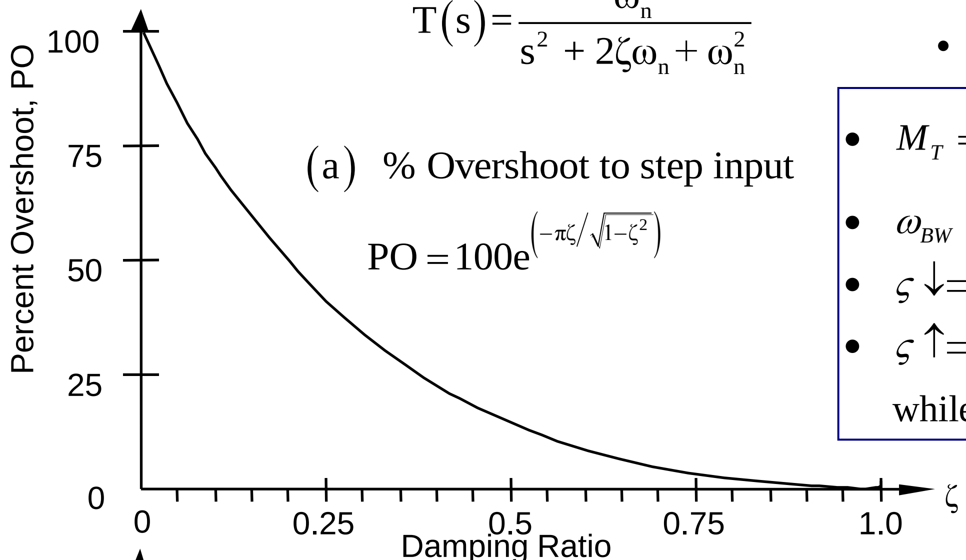
- Figures of merit:
  - PO = % overshoot = $100e^{-\pi\zeta/\sqrt{1-\zeta^2}}$ $\quad \zeta \leq 1$ As $\zeta \to 0$ system response becomes more oscillatory
  - t_r = 10% to 90% rise time $\approx 2.5/\omega_n$
  - Settling time = time to get and stay within $\pm$ x % of ss

$$TS_{5\%} \approx 3/\zeta\omega_n; \;\; TS_{1\%} \approx 4.7/\zeta\omega_n \quad \left(\zeta\omega_n = \text{time constant}^{-1}\right)$$

- "Think" in terms of nominal continuous (s-plane) pole locations given PO and TS specifications.
- Use LHP ‘ unit circle (s → z) map diagram to obtain desired pole locations in z-plane.

Results for 2nd Order Continuous System (or Dominant Pair of Complex Roots)

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$



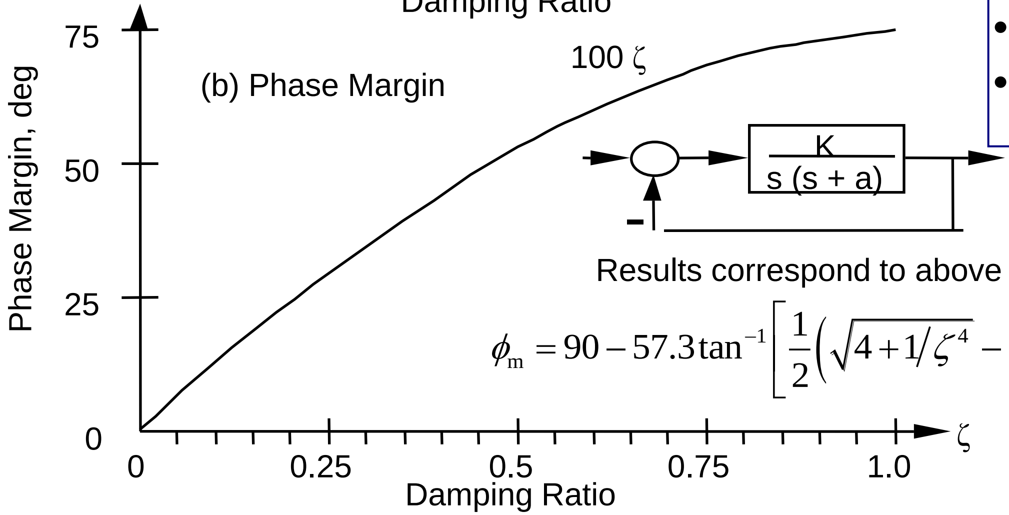
(a) % Overshoot to step input

$$PO = 100e^{\left(-\pi\zeta/\sqrt{1-\zeta^2}\right)}$$

- Relation to frequency domain measures

- $M_T = \| T(e^{j\omega h}) \|_\infty \le \sum_{k=0}^{\infty} | g(k) | = TV$
- $\omega_{BW} \propto 1/t_r$ (larger BW $\Rightarrow$ shorter rise time)
- $\varsigma \downarrow \Rightarrow M_T \uparrow$ & % OS $\uparrow$ and vice versa
- $\varsigma \uparrow \Rightarrow$ peak freq (time) moves to left (right) while peak moves to left and vanishes at $\varsigma = 1/\sqrt{2}$

- Automobile weight = 1000 kg; per wheel: 250kg
- Equivalent stiffness at each wheel = 60000 N/m
- What is damping constant, b to get 1%overshoot
- $\varsigma = 0.83 \Rightarrow b = 2\varsigma\sqrt{km} = 6430 N/m/s$



Results correspond to above loop structure:

$$\phi_m = 90 - 57.3\tan^{-1}\left[\frac{1}{2}\left(\sqrt{4 + 1/\varsigma^4} - 2\right)^{1/2}\right]$$

4a - Sensitivity

$$S_x^y = \frac{\%\text{ change in some } y(x)}{\%\text{ change in } x} = \frac{\Delta y/y}{\Delta x/x} \sim \frac{x}{y}\frac{\partial y}{\partial x}$$

Example: $y = f(x_1, x_2, .., x_n)$

$$\delta y = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}\,\delta x_i$$

$$\frac{\delta y}{y} = \sum_{i=1}^{n} \underbrace{\frac{\partial f}{\partial x_i}\frac{x_i}{y}}_{S_{x_i}^f}\,\frac{\delta x_i}{x_i}$$

$$\frac{\delta y}{y} = \sum_{i=1}^{n} S_{x_i}^f\,\frac{\delta x_i}{x_i}$$

- Series Compensation (SISO) $T(z) = \tilde{G}(z)H(z) \;/\; [\,1 + \tilde{G}(z)H(z)]$

$$S_{\tilde{G}(z)}^{T(z)} = \frac{\tilde{G}(z)}{T(z)}\frac{\partial}{\partial \tilde{G}(z)}\left[\frac{\tilde{G}(z)H(z)}{1+\tilde{G}(z)H(z)}\right] = \frac{1}{1+\tilde{G}(z)H(z)} = [1+L(z)]^{-1}$$

Return difference (RD) $\triangleq 1 + \tilde{G}(z)\,H(z) = 1 + L(z)$

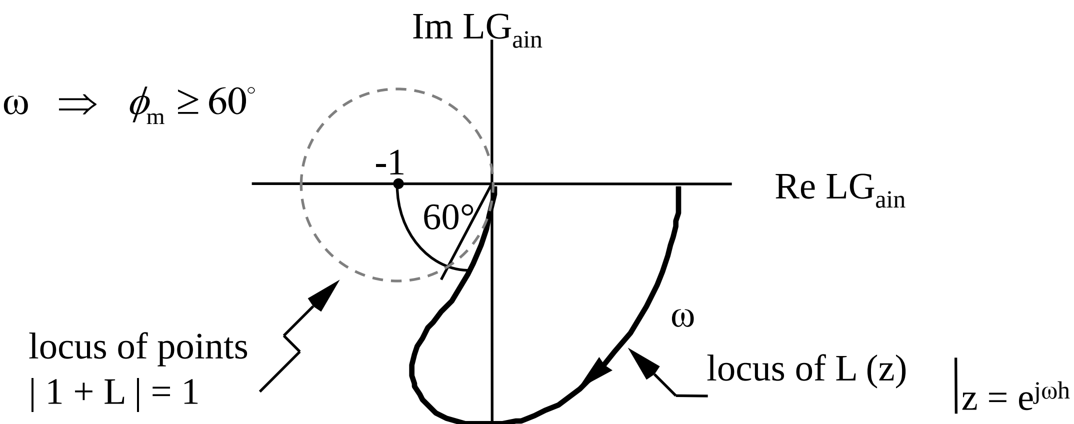
- SVFB

Return difference matrix $= I_m + K(zI - \Phi)^{-1}\,\Gamma$

Criteria: Keep | RD | >> 1 over frequency range of interest => large loop gain

- Relation to ϕ_m :

if $|RD| \geq 1$ for all $\omega \;\Rightarrow\; \phi_m \geq 60^\circ$



- Best to examine root locus of CL system poles with respect to individual parameter variations about their nominal values [a_i, b_i in G(s); a_{ij}, b_i in A, B; etc.]

5 - Choosing a Sensor

- **Environmental Factors**
  - Temperature Range
  - Humidity Effects
  - Corrosion
  - Size
  - Over Range Protection
  - Susceptibility to EM Interference
  - Ruggedness
  - Power Consumption
  - Self-test Capability
- **Economic Factors**
  - Cost
  - Availability
  - Mean-time-to-Failure
- **Sensor Rating Parameters**
  - Sensitivity
  - Dynamic Range
  - Resolution, Accuracy and Precision
  - Linearity
  - Zero Drift and Full-scale Drift
  - Useful Frequency Range and Bandwidth
  - Input and Output Impedance

5 – Sensor Rating Parameters - 1

- Sensitivity
  - Incremental output/ Incremental input = *dy/dx*
  - Example: Piezoelectric accelerometer sensitivities are measured in terms of picocoulombs (pC)/unit acceleration (g) or mv/g
- Dynamic range in dB
  - Lower limit = resolution of sensor
  - Dynamic range = range of operation/resolution

Sensor with a 16 bit ADC:

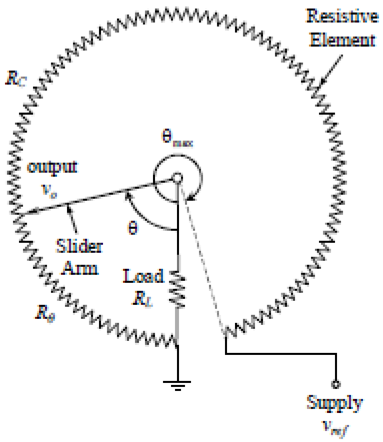
Dynamic range $= 20\log_{10}\left(2^{16}-1\right) = 96.3dB$

- Resolution : smallest change that can be detected/measured
  - Example: Required resolution for robot motion = 0.1 cm
  - Drive wheel of the robot directly driving a rotary potentiometer(pot) has diameter = 20cm
  - Assume diameter of pot = 10cm; Resistance, R = 5Ω; Resistivity of wire, ρ = 4μΩcm
  - Required resolution of the potentiometer, r = 0.1/20π = 0.0016
  - Number of turns, N = 1/0.0016 = 625
  - Wire diameter from circumference: π. 10=625 d

    $\Rightarrow d$ = 0.05cm =0.5mm
  - Diameter of the core of the coil, D from

$$R = \frac{\rho N(\pi D)}{\pi (d/2)^2}$$

$$\Rightarrow D = 1.25\text{cm}$$



5 – Sensor Rating Parameters - 2

- Linearity
  - How close output versus input curve is to a straight line under steady-state conditions
  - Linearity = (max deviation from the static calibration curve/Full scale value)*100%

– Zero Drift and Full-scale Drift
  - Causes of drift: sensor parameter changes (aging, wear and tear, nonlinearities, amplifier gain), ambient changes (temperature, pressure, humidity, vibration level), changes in power supply (ac line voltage, dc reference voltage)
  - Zero/Full-scale drift: changes in/stability of null (full-scale) reading

- Useful Frequency Range and Bandwidth
  - Typically ¼ the resonant frequency where gain is flat and phase is zero
  - Measure of sensor bandwidth

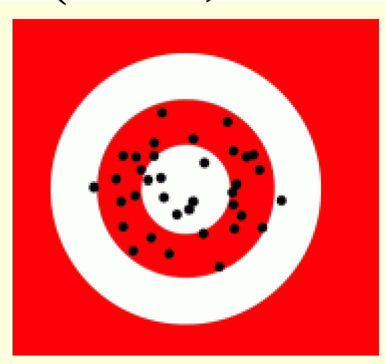
- Input and output impedance
  - Ratio of rated voltage/ current at the input port with output port open (no load)
  - Ratio of rated voltage/current at the input port when output port is shorted
  - Need isolation amplifiers when the output impedance is low

6- Accuracy versus Precision

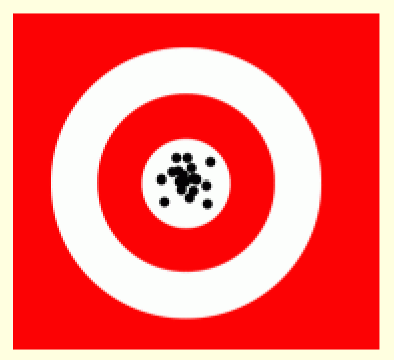
Neither Precise
Nor Accurate (bias and variance)

Precise, but not
Accurate (bias, small variance)

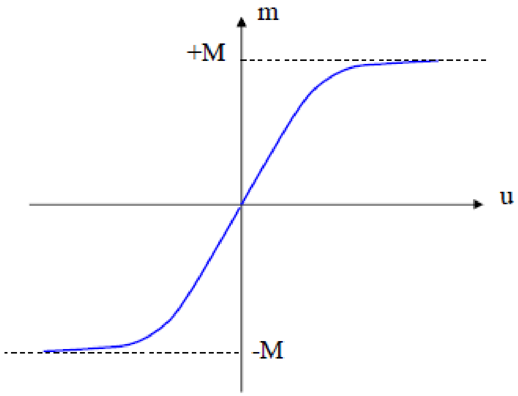
Not Precise, but reasonably
Accurate (no bias, some variance)



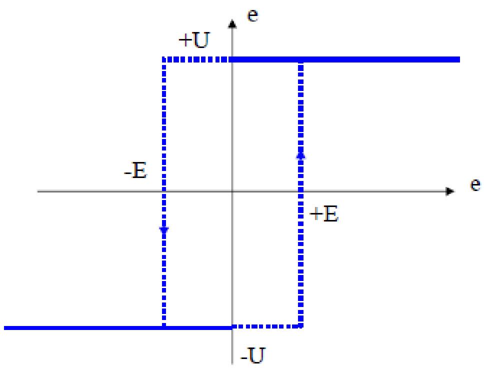
Precise and
Accurate (low bias and low variance)



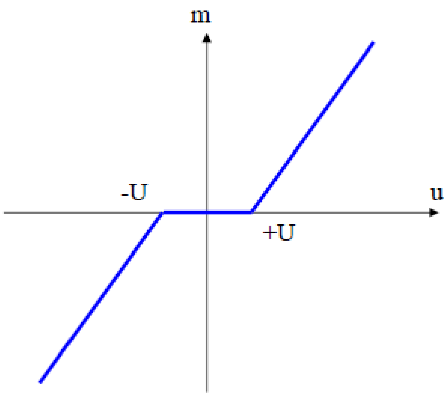
7- Actuator Nonlinearities



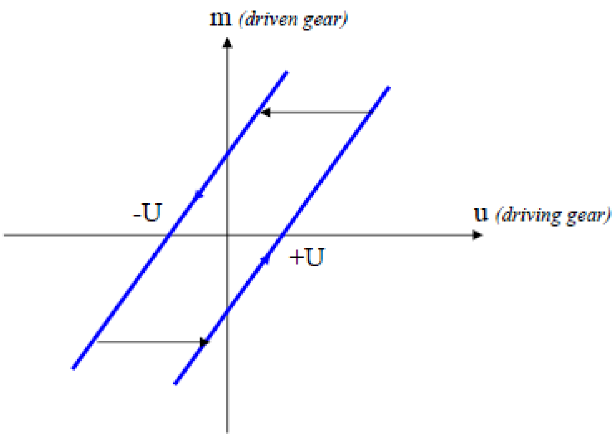
Saturation



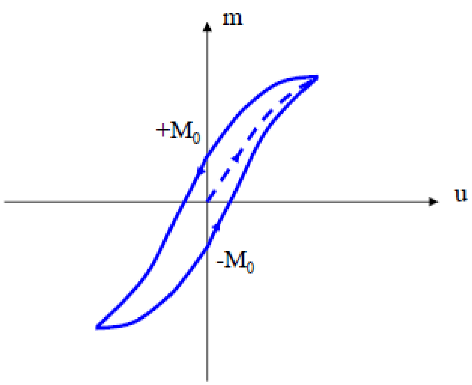
Relay Control (with Hysteresis)



Dead Zone



Gear Backlash



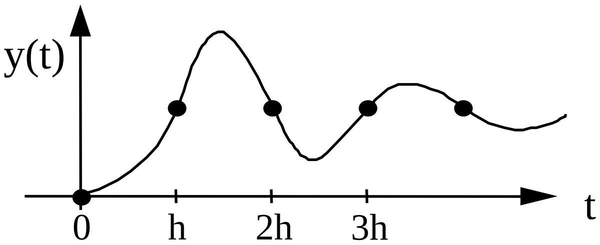
Hysteresis in Magnetic Materials

8 – Bandwidth Design

- Step 1: Decide on max frequency of operation ω_0 for the system based on response time requirements (BW)
  - A good rule of thumb: Cross-over frequency of loop gain is a good measure of BW
  - Another good rule of thumb: Rise time = 2.2/ ω_0
- Step 2: Design/ select relevant system components that have the capacity to operate at ω_0
- Step 3: Select feedback sensors with flat frequency response (operating frequency range) > 4 ω_0
- Step 4: Make sure that digital control computation can see at least 2 sensor samples per cycle … two-rate sampling (control sampling interval, *h* and sensor sampling interval, *h*/2)
- Step 5: Select signal conditioning and actuator system with flat frequency spectrum > ω_0
- Step 6: Integrate and test system performance. If performance specs are not met, make design changes and repeat again

C – Evaluation and Simulation

- Most time and effort is involved here!
  - To what extent have design specs been met
  - Actual ϕ_m
  - Closed-loop pole locations
  - Effect of different sample times, h
  - Computational lag
  - Root locus with respect to design parameters
- Time response of CL system to representative command inputs r(t) and initial conditions
  - Via computer simulations
  - Must consider response of y(t), $\underline{x}$(t) not only at the sample points, t = kh, but in between samples too!



(a pathological, but not far-fetched case)

- "What if" questions
  - Sensitivity of performance to changes in system parameters, controller parameters
  - Failure modes
  - Control saturation
  - Noise: measurement and/or process
  - Unmodelled dynamics, time-delays, ...
  - Quantization and other nonlinearities

  ⋮

Simulation of Closed-loop Time Response

- Tool to examine time response
  - Input ($\underline{u}$), output ($\underline{y}$), any state ($\underline{x}$)
  - Obtain response between sample points of the continuous-time variables $\underline{y}(t)$, $\underline{x}(t)$
  - $\underline{u}(t)$ is assumed piecewise constant over intervals of length h
  - Simulate with arbitrary initial conditions (user input)
  - Examine response to representative $\underline{r}(t)$

- Need a flexible computer program
  - Ability to input system dynamics in G(s) or in $\dot{\underline{x}} = A\underline{x} + B\underline{u}$, $y = C\underline{x} + (D\underline{u})$ format

  => program will work with a state-space model or TFM.

  If G(s) format given, get

  (i) SOF or SCF for SISO systems

  (ii) SCF for SIMO systems

  (iii) SOF for MISO systems

  (iv) Balanced minimal realization

  - Ability to simulate different control algorithms

    OPT = 0: Open-loop response $u(kh) = K_r\, r(kh)$

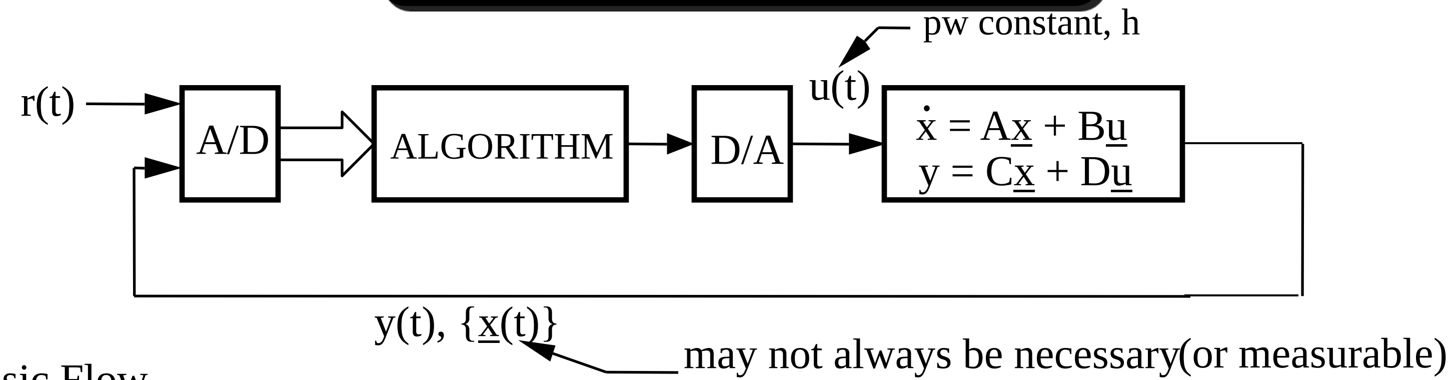
    OPT = 1: State variable feedback control

    OPT = 2±: Series compensation via H(z)(including different implementations)

    OPT = i, j, ... : Reserve for future control options

  - Ability to easily change the control interval, h

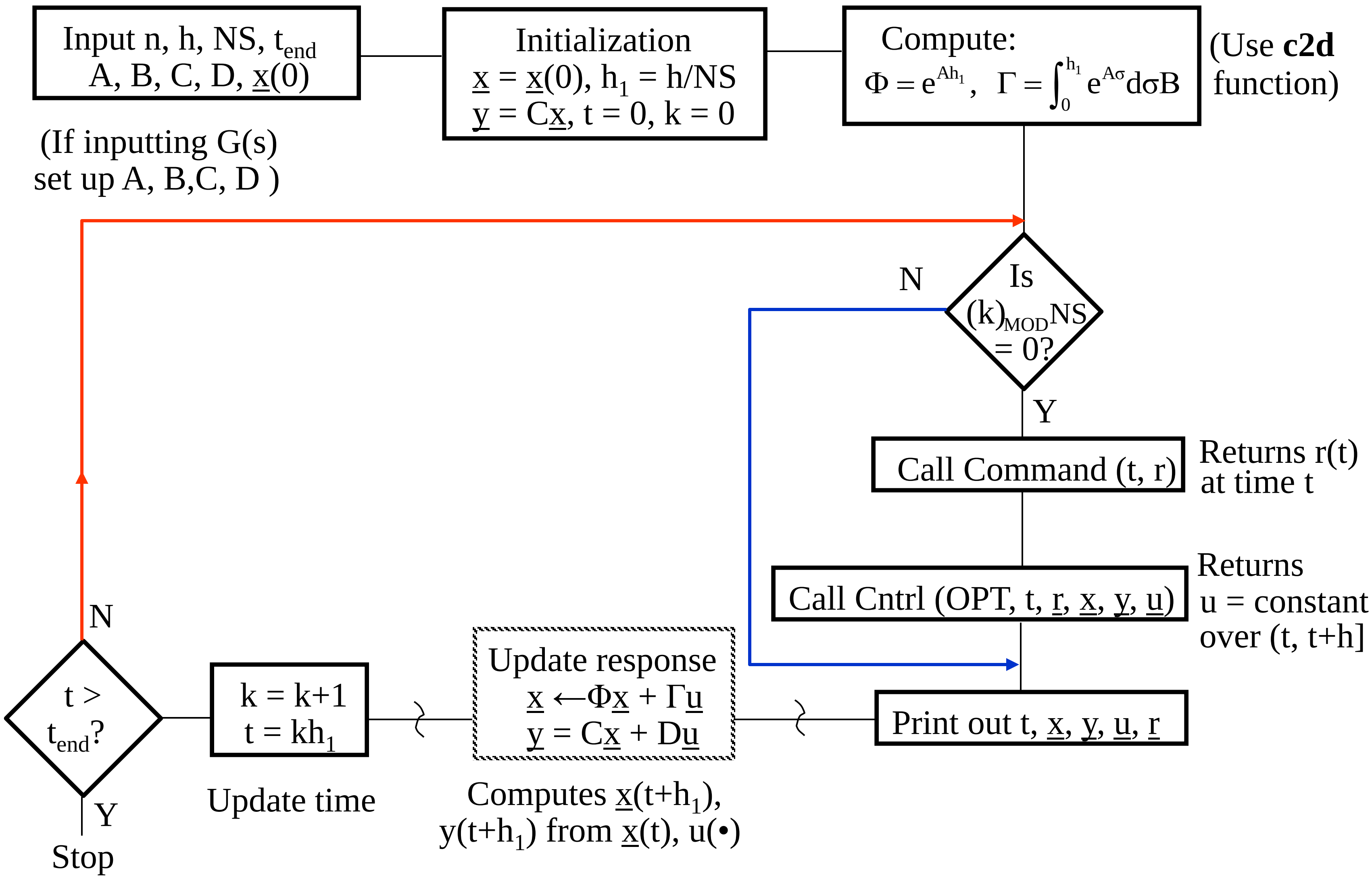
Simulation Structure



- Basic Flow
  1. Obtain r(t) at time t
  2. Sample r(t), y(t), $\underline{x}(t)$ at t = kh
     - Supply r(kh), y(kh), $\underline{x}(kh)$ to control algorithm
  3. Obtain u(kh) from control algorithm
     - $u(t) = u(kh)$ for $kh < t \leq (k+1)h$
  4. Print out info at time t: $\underline{x}$, y, u, r
  5. Compute system response $\underline{x}(t)$, y(t) over (kh, (k+1)h]
     - e.g., at t = (k+1)h: $\underline{x}[(k+1)h] = \Phi(h)\,\underline{x}(kh) + \Gamma(h)\,u(kh)$
- How to compute $\underline{x}(t)$ and y(t) at more points in [kh, (k+1)h]
  - Pick $NS \geq 1$ and let $h_1 = h/NS$
  - The control algorithm is active every h sec ($\underline{u}$ is piecewise constant over intervals of length h)
  - Can compute $\underline{x}(t)$ at times that are multiples of h_1, while changing u every NS-th multiple of h_1
  - Dual-time scale simulation (NS = 2 → 5 usually)

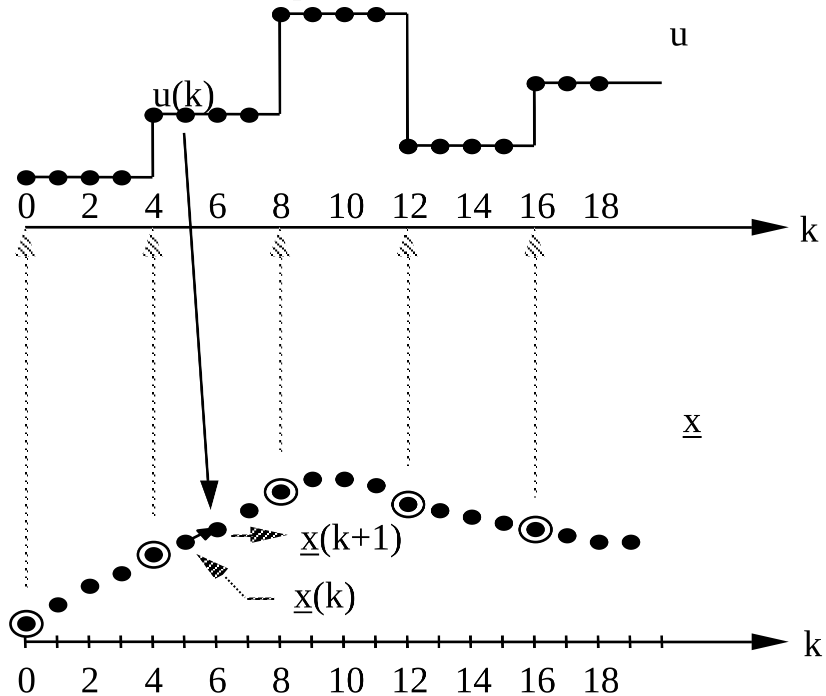
[Remember!! - even though we simulate the system response using a (small) time step h_1, the control algorithm must have been designed for the actual sample time h.]

Flow Diagram for Simulation Program



Time History of Simulated Sequences (Example, NS=4)

- An understanding facilitates subsequent simulations that will include time-delay



1. New control computed only at times k = 0, 4, 8, 12, ... using the corresponding value of $\underline{x}$ (or $\underline{y}$) at this time. The value of $\underline{u}$ is not changed at other than these points.
2. Next $\underline{x}(k+1)$ is computed at time k, k = 0, 1, ... using $\underline{x}(k)$ --- the previous $\underline{x}$ and current $\underline{u}$.

   This computation is done at every k.

$$\underline{x}\,[(k+1)h_1] = \Phi\underline{x}(kh_1) + \Gamma\underline{u}(kh_1)$$

$$\underline{y}\,[(k+1)h_1] = C\underline{x}\,[(k+1)h_1] + D\underline{u}(kh_1)$$

Control Algorithm Simulation

- Command (t, $\underline{r}$) and Cntrl (OPT, t, $\underline{r}$, $\underline{x}$, $\underline{y}$, $\underline{u}$) are user-oriented.
- Command (t, $\underline{r}$) returns $\underline{r}$(t), e.g., $r_i = 1$, $r_i = A_i * t$, etc.
- Cntrl must distinguish among various options:

<u>OPT = 0</u> for open-loop response, $\underline{u} = K_r \underline{r}$

<u>OPT = 1</u> for SVFB, $\underline{u} = K_r \underline{r} - K\underline{x}$

$$\underline{u} = K_r \underline{r} - \sum_{i=1}^{n} \underline{k}_i * x_i ; \underline{k}_i = col\ i\ of\ K$$

where the gain values K_r, $\{\underline{k}_i\}$ are read in as input or else set via an input statement.

<u>OPT = 2</u> for "standard" series compensation (q-th order). Read coefficients of TFM *H(z)*

$\underline{u}(z) = H(z)\underline{e}(z) \Rightarrow u_i(z) = \sum_{j=1}^{p} h_{ij}(z) e_j(z);\ i = 1, 2, \ldots, m.$ Scale each row of $H(z)$ so that it has the same least common denominator of order $q_i \Rightarrow u_i(z) = \sum_{j=1}^{p} h_{ij}(z) e_j(z) = \dfrac{\sum_{j=1}^{p}\sum_{l=0}^{q_i} \beta_{ijl} z^{-l} e_j(z)}{1 + \sum_{l=1}^{q_i} \alpha_{il} z^{-l}}; i = 1, 2, .., m$

- Corresponding discrete algorithm:

$$u_i(k) = \sum_{j=1}^{p} \beta_{ij0} e_j(k) + \underbrace{\sum_{j=1}^{p}\sum_{l=1}^{q_i} \beta_{ijl}\, e_j(k-l)}_{SE_i} + \underbrace{\sum_{l=1}^{q_i} \alpha_{il} u_i(k-l)}_{SU_i}; i = 1, 2, .., m$$

- To implement $\underline{u}$(k) via H(z) will need (m.p.(q+1)) storage (each $q = \max_i q_i$) for the last q values of each $\underline{e}$ and $\underline{u}$: paste, pastu.

There will be other options to cover different implementations.

Algorithm Flow for Implementing H(z) Compensator

0 - enter with t, r, y

1 - if t = 0, set pastu(k,i) = 0, k=1,2,..,q; i=1,2,..,m; paste(k,j) = 0 for j=1,2,…,p;k=1,2,..,q

SE(i) = 0, SU(i) = 0, S(i) = 0 for i=1,2,..,m

2 - e(j) = r(j) – y(j) for j=1,2,..,p

3 - for i=1,m

u(i) = S(i)

for j=1,p

$u(i)=u(i)+\beta_{ij0}e(j)$ (This is the new value of u(i) we are computing)

end

end

4 - (pushdown pastu, paste, if q > 1)

for i = 1, m & for k=1,q-1 pastu (q + 1 – k,i) = pastu (q – k,i) end i and end k

for j = 1, p & for k=1,q-1 paste (q + 1 – k,j) = paste (q – k,j) end j and end k

Setup for next time through.

5 - for i=1,m pastu (1,i) = u(i) (Store latest u)

for j=1,p paste (1,j) = e(j) (Store latest e)

6 - $SE(i) = \sum_{j=1}^{p} \sum_{l=1}^{q_i} \beta_{ijl}\, paste(l, j)$

$SU(i) = \sum_{l=1}^{q_i} \alpha_{il}\, pastu(l,i)$

S (i) = SE (i) – SU (i)

7 - return

This implementation of H(z) is not the best from a numerical accuracy viewpoint, especially for *q*>2. Use partial Fraction expansion.

- Special case when q = 1 and SISO:
  e = r – y
  $u = \beta_0 e + S$
  $S = \beta_1 e - \alpha_1 u$

- Try to program Cntrl in much the same way for as would be done in the real-time implementation. (Note, u can be output at step 3.)
- Such an implementation permits timing of code, investigation of round-off effects, testing, etc.

Simulation of Time Delay, τ

- Lump all delay in the control (input lag)

$$\dot{\underline{x}} = A\underline{x} + \sum_{j=1}^{m} \underline{b}_j u_j(t-\tau_j); \underline{b}_j = \text{ column } j \text{ of } B$$

$$\underline{y} = C\underline{x} + \sum_{j=1}^{m} \underline{d}_j u_j(t-\tau_j); \underline{d}_j = \text{ column } j \text{ of } D$$

- Write delay as multiple of simulation step, h_1

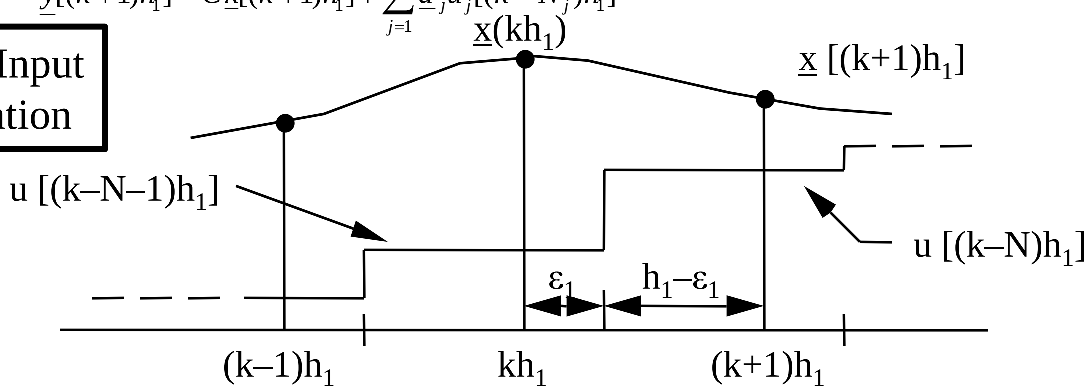
$$\tau_j = N_j h_1 + \varepsilon_{1j}, \quad 0 \le \varepsilon_{1j} \le h_1$$

- Discrete simulation model

$$\underline{x}[(k+1)h_1] = \Phi \underline{x}[kh_1] + \sum_{j=1}^{m} [\underline{\gamma}_{1j} u_j[(k-1-N_j)h_1] + \underline{\gamma}_{0j} u_j[(k-N_j)h_1]]$$

$$\underline{y}[(k+1)h_1] = C\underline{x}[(k+1)h_1] + \sum_{j=1}^{m} \underline{d}_j u_j[(k-N_j)h_1]$$

Single Input illustration



$$\Phi = e^{Ah_1}$$

$$\underline{\gamma}_{0j} = \int_0^{h_1-\varepsilon_{1j}} e^{A\sigma} d\sigma \underline{b}_j$$

$$\underline{\gamma}_{1j} = \int_{h_1-\varepsilon_{1j}}^{h_1} e^{A\sigma} d\sigma \underline{b}_j$$

$$= e^{A(h_1-\varepsilon_{1j})} \int_0^{\varepsilon_1} e^{A\sigma} d\sigma \underline{b}_j$$

- Will need an (N_j+2)-vector pushdown stack to store past values of u_j, uold(i,j), i = 1, ... , N_j+1, j=1,2,..,m and latest value uold(N_j+2,j). $U_{old}(i,j) \equiv u(k-N_j-2+i,j)$ Initialize $u_{old}(i,j) = 0$ at t = 0.

- Control algorithm design is based on delay model, Eq. (2.34), associated with time step h.

Required Modifications to Simulation Flow Diagram

- Initialization:

  compute $N_j = \text{Int}\,[\tau_j / h_1]$

  $\varepsilon_{1j} = \tau_j - N_j\, h_1$

  set $u_{old}(i,j) = 0$ for $i = 1, \ldots, N_j+2;\ j=1,2,..,m$

  compute $\underline{\gamma}_{0j}, \underline{\gamma}_{1j};\ j=1,2,..,m$

  (Note, $\underline{\gamma}_{0j} = 0$ if $\varepsilon_{1j} = h_1^-$, $\underline{\gamma}_{1j} = 0$ if $\varepsilon_{1j} = 0$)

- New Response Update Module

$u_{old}(N_j+2,j) = u(j);\ j=1,2,..,m$

↓

Propagate $\underline{x}$, y

$$\underline{x} \leftarrow \Phi \underline{x} + \sum_{j=1}^{m} [\underline{\gamma}_{1j} u[2,j] + \underline{\gamma}_{0j} u[1,j]]$$

$$\underline{y} = C\underline{x} + \sum_{j=1}^{m} \underline{d}_j u(2,j)$$

Computes $\underline{x}(t+h_1), \underline{y}(t+h_1)$

↓

Pushdown $u_{old}(i,j)$
(only if $\tau_j > 0$)
$u_{old}(i,j) = u_{old}(i+1,j)$
for $i = 1, 2, \ldots, N+1$

↓

- $\underline{u}_{old}$ stack will be piecewise constant values that change every NSth point.
- Correctly simulates small delay (when $N_j = 0$, i.e., $\tau_j < h_1$).

Summary

1. Design Approach and the Design Process

2. Performance Measures and Criteria
    - Stability and phase margin
    - Steady-state accuracy
    - Max peak criteria
    - Speed of response/transient, sum of absolute error, sum of square error
    - Sensitivity and return difference
    - Sensor rating parameters
    - Actuator nonlinearities
    - Bandwidth design

3. Simulation of Closed-Loop Time Response
    - Simulation program structure
    - Control algorithm simulation
    - Modifications to simulate time delay

\